

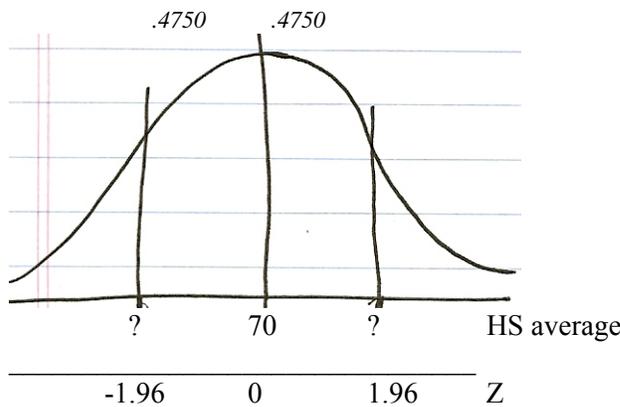
SOLUTION TO HW: THE NORMAL DISTRIBUTION

PROBLEM 1:

Suppose that New York State high school average scores, for students who graduate, are normally distributed with a population mean of 70 and a population standard deviation of 13.

- a) The “middle” 95% of all NYS high school students have average scores between _____ and _____ ?
- b) What proportion of NYS high school students have average scores between 60 and 75?
- c) Calculate the 14th percentile.
- d) Calculate the 92nd percentile.

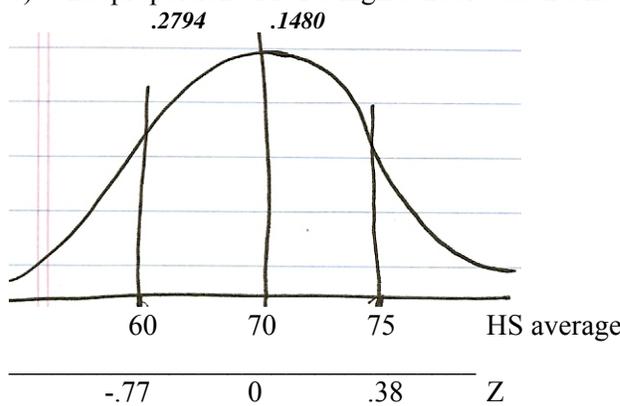
- a) The “middle” 95% of all NYS high school students have average scores between ___ and ___ ?



$$70 \pm 1.96 (13) \rightarrow 70 \pm 25.48$$

ANS: 95% of all NYS high school students have average scores between 44.52 and 95.48.

- b) What proportion of NYS high school students have average scores between 60 and 75?

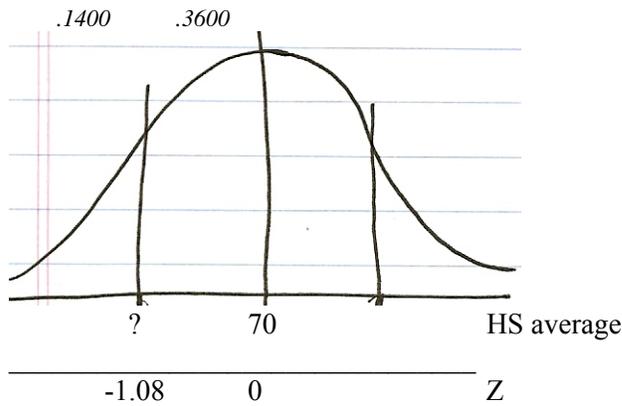


$$Z = (60 - 70) / 13 = -0.77; \quad Z = (75 - 70) / 13 = .38$$

$$\text{ANS: } .2794 + .1480 = .4274$$

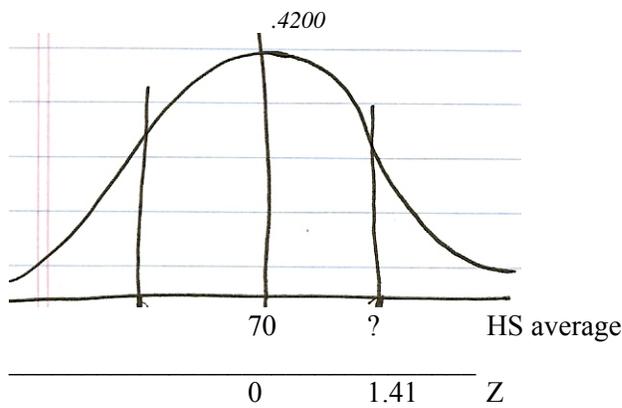
42.74% of NYS high school students have average scores between 60 and 75.

c) Calculate the 14th percentile.



$$Z = -1.08 = (X - 70) / 13 \quad \boxed{X = 55.96 \text{ ANS}}$$

d) Calculate the 92nd percentile.

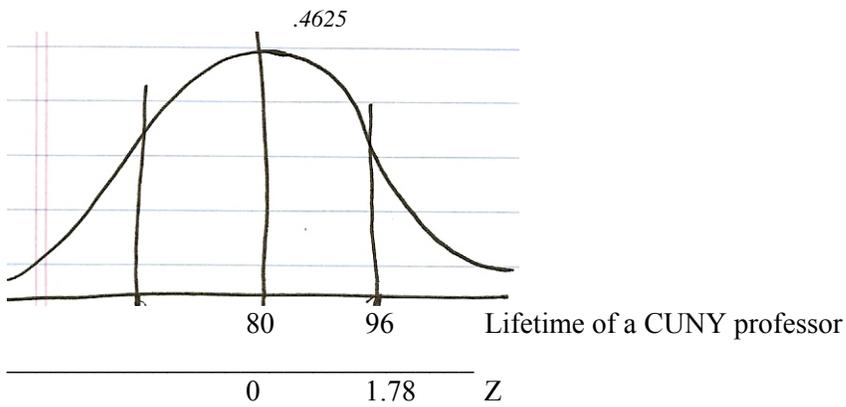


$$Z = 1.41 = (X - 70) / 13 \quad \boxed{X = 88.33 \text{ ANS}}$$

PROBLEM 2 :

Suppose CUNY professors have an average life, normally distributed, of 80 years with a population standard deviation of 9 years.

a) What percent of CUNY professors will live more than 96 years?

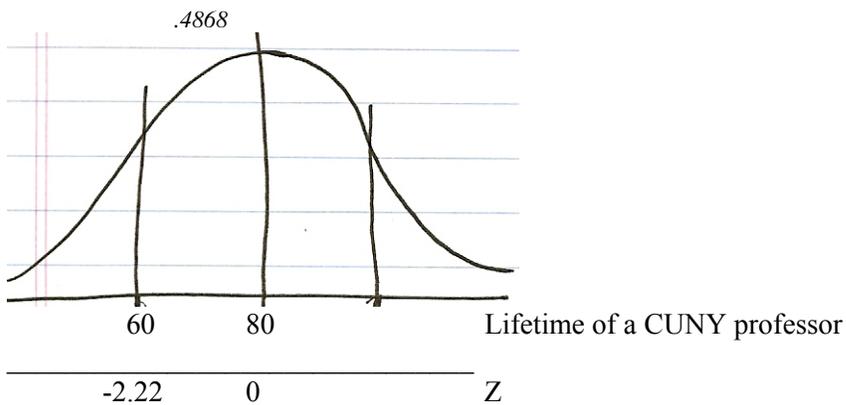


$$Z = (96 - 80) / 9 = 1.78$$

$$\text{ANS: } .5 - .4625 = .0375$$

3.75% of CUNY professors will live more than 96 years.

b) What percent of CUNY professors will not make it past the age of 60?

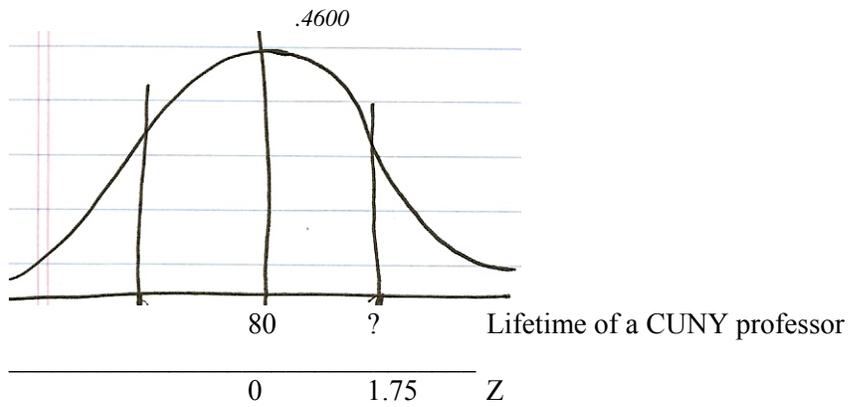


$$Z = (60 - 80) / 9 = -2.22$$

$$\text{ANS: } .5 - .4868 = .0132$$

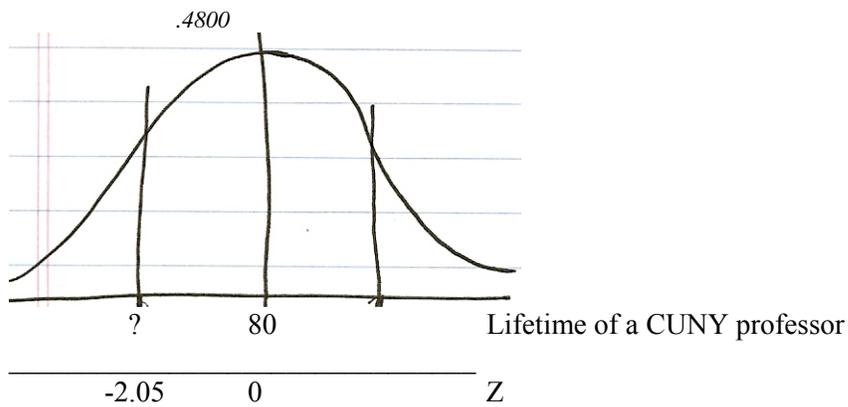
1.32% of CUNY professors will not live beyond age 60.

c) Calculate the 96th percentile.



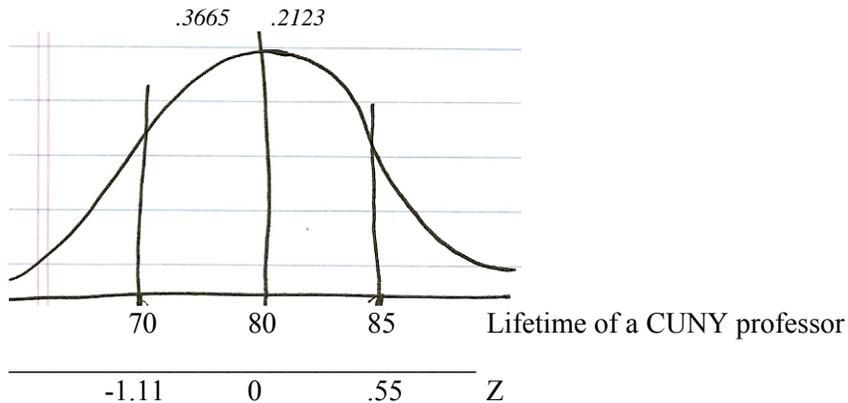
$$Z = 1.75 = (X - 80) / 9 \quad \boxed{X = 95.75 \text{ years :ANS}}$$

d) Calculate the 2nd percentile.



$$Z = -2.05 = (X - 80) / 9 \quad \boxed{X = 61.55 \text{ years :ANS}}$$

e) What proportion of CUNY professors will live between 70 and 85 years?



$$Z = (70 - 80) / 9 = -1.11$$

$$Z = (85 - 80) / 9 = 0.55$$

$$\text{ANS: } .3665 + .2123 = .5788$$

57.88% of CUNY professors will live between 70 and 85 years

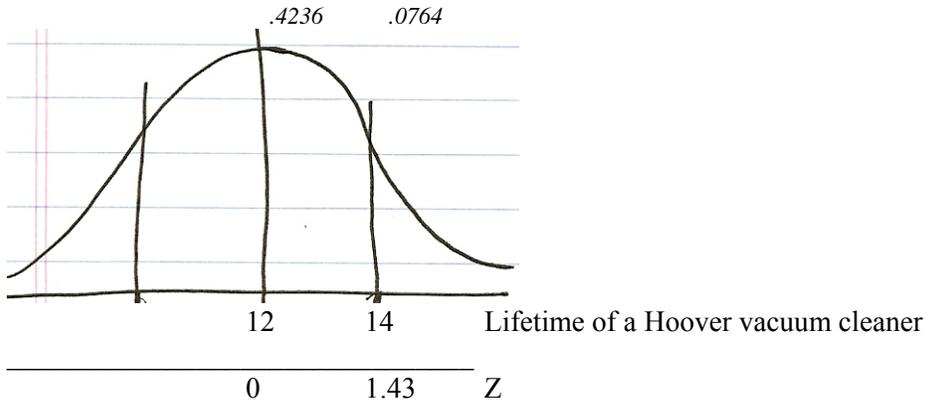
PROBLEM 3:

Suppose the lifetimes of Hoover vacuum cleaners are normally distributed with an average life (μ) of 12 years and a population standard deviation (σ) of 1.4 years.

- What proportion of Hoover vacuum cleaners will last 14 years or more?
- What proportion of Hoover vacuum cleaners will last 9 years or less?
- What proportion of Hoover vacuum cleaners will last between 11 and 13 years?
- Calculate the 80th percentile.
- Calculate the 7th percentile.

Suppose the lifetimes of Hoover vacuum cleaners are normally distributed with an average life (μ) of 12 years and a population standard deviation (σ) of 1.4 years.

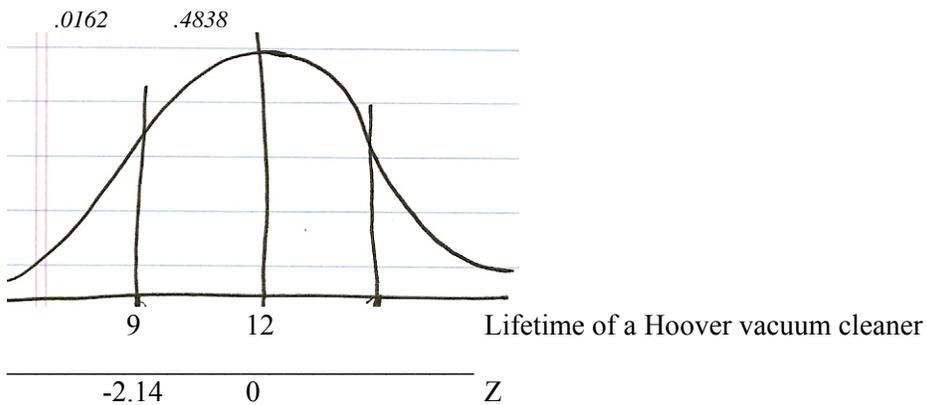
- What proportion of Hoover vacuum cleaners will last 14 years or more?



$$Z = (14 - 12) / 1.4 = 1.429$$

ANS: 7.64% of Hoover vacuum cleaners will last 14 years or more.

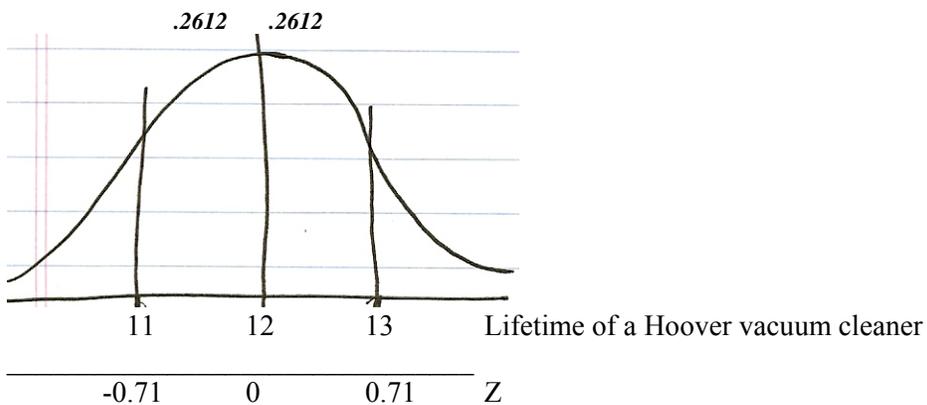
b) What proportion of Hoover vacuum cleaners will last 9 years or less?



$$Z = (9 - 12) / 1.4 = -2.14$$

ANS: 1.62% of Hoover vacuum cleaners will last 9 years or less.

c) What proportion of Hoover vacuum cleaners will last between 11 and 13 years?



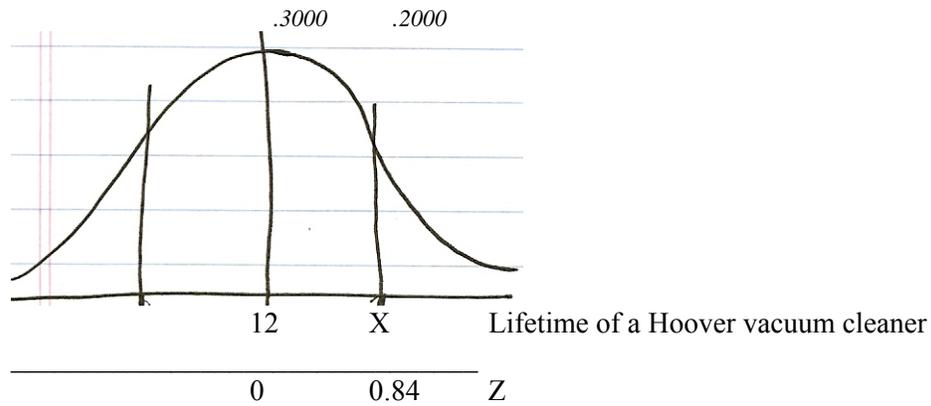
$$Z = (13 - 12) / 1.4 = .714$$

$$Z = (11 - 12) / 1.4 = -.714$$

$$\text{ANS: } .2611 + .2611 = .5222$$

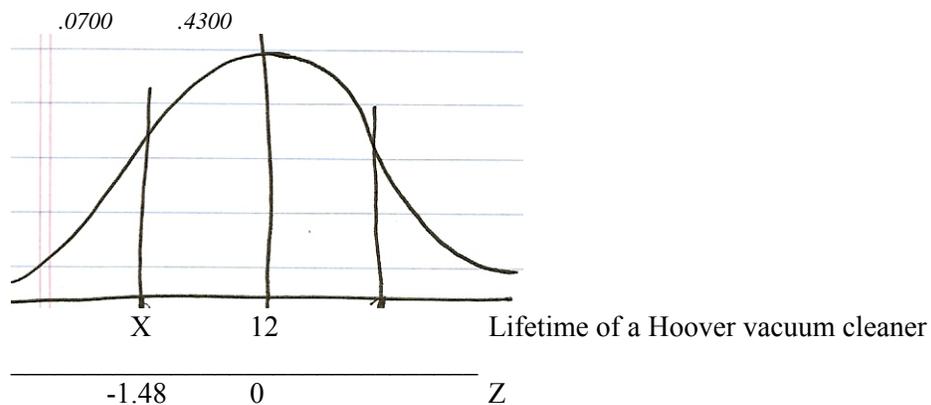
52.22% of Hoover vacuum cleaners will last between 11 and 13 years.

d) Calculate the 80th percentile.



$$Z = 0.84 = (X - 12) / 1.4 \rightarrow \boxed{X = 13.176 \text{ years} : \text{ANS}}$$

e) Calculate the 7th percentile.



$$Z = -1.48 = (X - 12) / 1.4 \rightarrow \boxed{X = 9.928 \text{ years} : \text{ANS}}$$

PROBLEM 4:

Scores of high school seniors taking the English Regents examination in New York State are normally distributed with a mean of 70 and a standard deviation of 10. Find the probability that a randomly selected high school senior will have a score between 70 and 75?

Ans.: .1915

PROBLEM 5:

Science scores for high school seniors in the United States are normally distributed with a mean of 60 and a standard deviation of 15. Students scoring in the top 3% are eligible for a special prize consisting of a laptop and \$5,000. What is the approximate cutoff score a student must get in order to receive the prize?

Ans.: 88.2

PROBLEM 6: Life of a GE Stove

The average life of a GE stove is 15.0 years (population mean) with a population standard deviation of 2.5 years.

- (a) What percentage of GE stoves will last 10 years or less? .0228
- (b) What percentage of GE stoves will last 18 years or more? .1151
- (c) What percentage of GE stoves will last between 16 and 20 years? .3218
- (d) Calculate the 1st percentile $Z = -2.33$ answer = 9.175 years
- (e) Calculate the 96th percentile $Z = +1.75$ answer = 19.375 years

PROBLEM 7: The average wage of plumbers

The average hourly wage of plumbers is normally distributed with a population mean of \$24.00 and a population standard deviation of \$6.00. Calculate the following:

- (a) The proportion of plumbers earning between \$18 and \$22
 $.3413 - .1293 = .2120$
- (b) The proportion of plumbers earning more than \$28 $.5000 - .2454 = .2546$
- (c) The proportion of plumbers earning less than \$15 $.5000 - .4332 = .0668$
- (d) The 70th percentile $Z = +.52$

$$.52 = (X - 24) / 6 \quad X = \$27.12$$