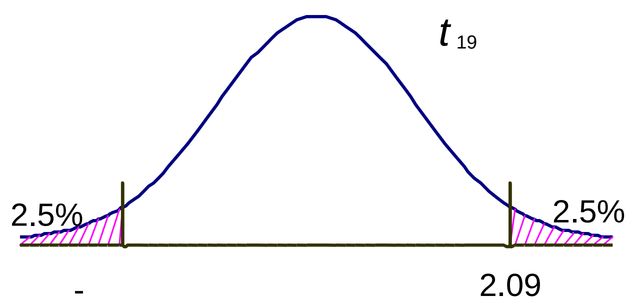
The One-Sample t DISTRIBUTION

SOLUTION TO HOMEWORK: STATISTICAL INFERENCE
THE t DISTRIBUTION

PROBLEM 1:
A company manufactures bars of soap that are supposed to weigh exactly (on average) 10 ounces. A sample is taken:

n=20
$\overline{X}$ = 10.20 oz
s = 0.80 oz

(a) Test at $\alpha = .05$
H_0: $\mu = 10$ oz
H_1: $\mu \neq 10$ oz



$$t_{19} = \frac{10.20 - 10.00}{\frac{.80}{\sqrt{20}}} = \frac{.20}{.18} = 1.11$$

DO NOT REJECT H_0

(b) Construct a 95% CIE of μ

$10.20 \pm 2.093\ (.18)$

$10.20 \pm .38$

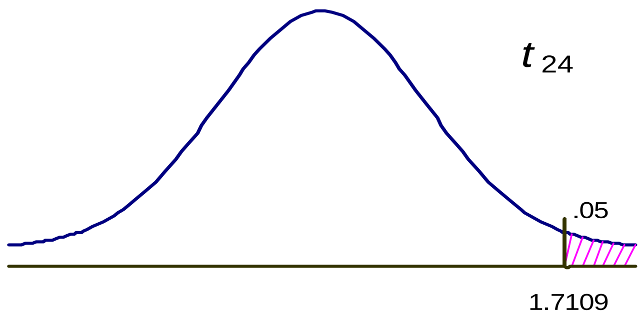
$9.82 \leftrightarrow 10.58$ oz

PROBLEM 2:
The Momoland Springs Company claims that at most there are 1 ppm (parts per million) of benzene in their water.

Test at $\alpha=.05$

Data:
n=25
$\overline{X}$ = 1.16 ppm
s = .20 ppm

H_0: $\mu \leq 1$ ppm
H_1: $\mu > 1$ ppm



$$t_{24} = \frac{1.16 - 1.00}{.20/\sqrt{25}} = \frac{.16}{.04} = 4$$

REJECT H_0

PROBLEM 3:
A company claims that cancer patients using drug X will live at least 10 more years.

n=16

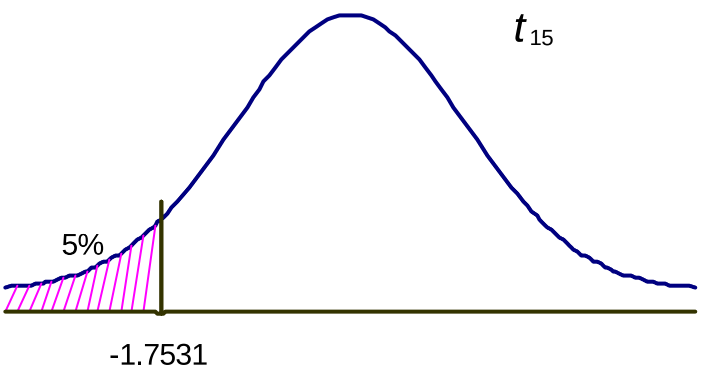
$\overline{X}$ = 8.8 year

s = 3.4 year

Test at α=.05

H_0: $\mu \geq 10$ year

H_1: $\mu < 10$ year



$$t_{15} = \frac{8.8 - 10}{3.4/\sqrt{16}} = \frac{-1.20}{.85} = -1.41$$

DO NOT REJECT H_0

PROBLEM 4:
A company manufactures wind turbines. A random sample of 25 turbines is taken and the sample mean life is 20.00 years with a standard deviation of 2.50 years. If you were constructing a 95% two-sided confidence interval estimate, the upper limit would be:

t_{24} is 2.0639 for the two-sided 95% confidence interval.

20.00 ± 2.0639 (2.50/√25)

20.00 ± .2.0639 (.50)

20.00 ↔ 1.03 years

Answer: 21.03 years

PROBLEM 5:
You want to estimate the average life of a new kind of computer chip. You take a sample of 16 chips and find the sample mean to be 12.50 years with a sample standard deviation of .80 years. You construct a 99% two-sided confidence interval estimate. It is:

t_{15} is 2.9467 for the two-sided 99% confidence interval.

$12.50 \pm 2.9467\ (0.80/\sqrt{16})$

$12.50 \pm 2.9467\ (.20)$

$12.50 \pm .59$

Answer: 12.50 ± .59 years

11.91 years ↔ 13.09 years

PROBLEM 6:

A yogurt company claims that its yogurt ice cream has no more than 2.00 mgs. of fat. You sample 25 cups of yogurt and find that the sample mean is 2.10 milligrams and the sample standard deviation is .40 milligrams

(a) Test the claim at the .05 significance level
(b) No claim was made by the company. They took a sample with the above results and want to construct a two-sided 95% confidence interval for the population mean. The upper limit of the confidence interval is:

(a) The critical T-value for a one-tail test at the .05 level: $t_{24} = +1.7109$
The calculated t-statistic is: $(2.10 – 2.00) / (.40 /\sqrt{25}) = .10 / .08 = 1.25$
Since 1.25 is not more than 1.7109, you cannot reject the null hypothesis. You may be looking at sampling error. The difference of .10 milligrams is not statistically significant.

(b)This we do as a two-sided CI. $2.10 \pm 2.0639\ (0.40/\sqrt{25})$
$2.10 \pm .17$

ANSWER = 2.27 MGS.

PROBLEM 7:

A school claims that the average English regents score of its students is at least 80. You sample 16 students and find that the sample mean is 76 and the sample standard deviation is 12

(a) Test the claim at the .05 significance level
(b) No claim was made by the school. They took a sample with the above results and want to construct a two-sided 95% confidence interval for the population mean. The upper limit of the confidence interval is:

(a) The critical T-value for a one-tail test at the .05 level: t_{15} = -1.7531
The calculated t-statistic is: (76 – 80) / (12 /√16) = -4 / 3 = -1.33
Since -1.33 is not less than -1.7109 (you are not in the rejection region), you cannot reject the null hypothesis. You may be looking at sampling error. The difference of 4 points is not statistically significant.

(b)This we do as a two-sided CI. 76 ± 2.1315 (12 /√16)
76 ± 6.39

ANSWER = 82.39

PROBLEM 8:

An LSAT preparation school claims that its review course will add at least 50 points to the score of a student retaking the LSAT exam. You sample 25 students and find that average improvement was 40 points with a standard deviation of 15 points

(a) Test the claim at the .05 significance level
(b) No claim was made by the school. They took a sample with the above results and ask you to construct a two-sided 95% confidence interval for the population mean.

(a) The critical T-value for a one-tail test at the .05 level: t_{24} = -1.7109
The calculated t-statistic is: (40 – 50) / (15 /√25) = -10 / 3 = -3.33
Since -3.33 is less than -1.7109 (you are in the rejection region), you reject the null hypothesis. The difference of 10 points is statistically significant.

(b)This we do as a two-sided CI. 40 ± 2.0639(15 /√25)

40 ± 6.19

33.81 ↔ 46.19

PROBLEM 9:

A drug company that manufactures a diet drug claims that those using the drug for 30 days will lose at least 15 pounds. You sample 30 people who have used the drug and find that the average weight loss was 12 pounds with a standard deviation of 5 pounds.
(a) Test the claim at the .05 significance level. (b) No claim was made by the company. They took a sample with the above results and ask you to construct a two-sided 95% confidence interval for the population mean.

(a) The critical T-value for a one-tail test at the .05 level: $t_{29} = -1.6991$
The calculated t-statistic is: $(12 – 15) / (5 /\sqrt{30}) = -3 / .91 = -3.30$
Since -3.30 is less than -1.6991 (you are in the rejection region), you reject the null hypothesis. The difference of 3 pounds is statistically significant.

(b)This we do as a two-sided CI. 12 ± **2.0452**(5 /√30)

12 ± 1.87

10.13 pounds ↔ 13.87 pounds

The drug company should not claim more than a 13.87 pound weight loss due to the drug.