

SOLUTIONS: STATISTICAL INFERENCE
One-Sample Z-test and Two-sided Confidence Interval Estimators Using Z

PROBLEM 1:

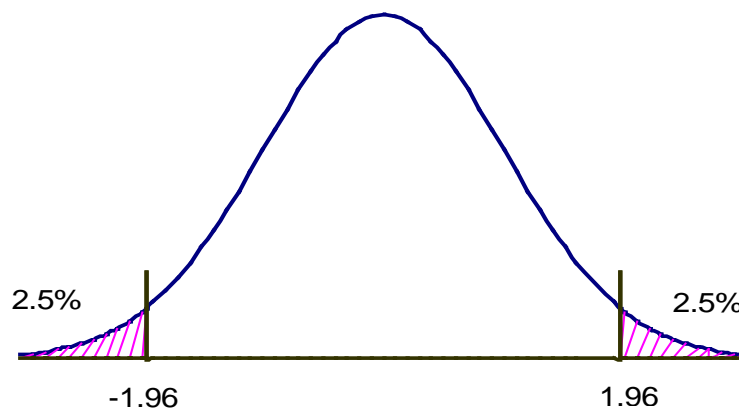
A company wishes to determine if the average salary of its clerks is really \$340. The company researcher takes a sample of 64 clerks and finds that $\bar{X} = \$300$ and $s = \$80$.

a) Test at $\alpha = 0.05$.

b) Construct a 95% CIE of μ

$$H_0: \mu = \$340$$

$$H_1: \mu \neq \$340$$



$$Z = \frac{300 - 340}{80 / \sqrt{64}} = \frac{-40}{10} = -4 \quad \text{REJECT } H_0$$

Therefore, reject H_0 at $p < .05$

(b) As a 95%, CIE:

$$300 \pm \frac{80}{\sqrt{64}} = 300 \pm 1.96(10) = 300 \pm 19.6$$

$$\$280.40 \longleftarrow \longrightarrow \$319.60$$

[NOTE: 340 is not in this interval.]

PROBLEM 2: A company manufactures batteries for watches. A random sample of 144 batteries is taken and the sample mean life is 5.5 years with a standard deviation of 0.6 years. If you were constructing a 99% two-sided confidence interval estimate, the lower limit would be:

Answer: 5.37 years

PROBLEM 3:

A company claims that its soup machines deliver exactly 10.0 ounces of soup—no more, no less. A researcher samples 100 bowls of soup and finds that:

$$\bar{X} = 10.40 \text{ ounces}$$

$$s = 1.20 \text{ ounces}$$

(a) Test the company's claim at a .05 significance level

$$H_0: \mu = 10.0 \text{ ounces}$$

$$H_1: \mu \neq 10.0 \text{ ounces}$$

ANSWER: Z = 3.33 Reject H₀; p < .05

PROBLEM 4:

A company claims that its toasters have an average life of at least 10 years. Use the following sample to test at $\alpha = .02$.

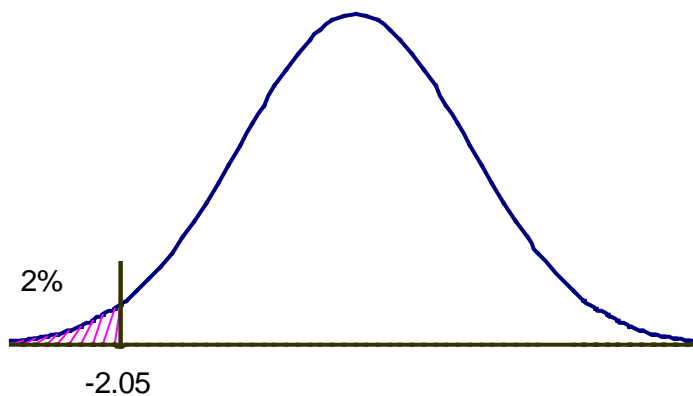
$$n=100$$

$$\bar{X} = 9.4 \text{ years}$$

$$s = 1.2 \text{ years}$$

$$H_0: \mu \geq 10 \text{ years}$$

$$H_1: \mu < 10 \text{ years}$$



$$Z = \frac{9.4 - 10}{1.2 / \sqrt{100}} = \frac{-.60}{.12} = -5$$

REJECT H₀

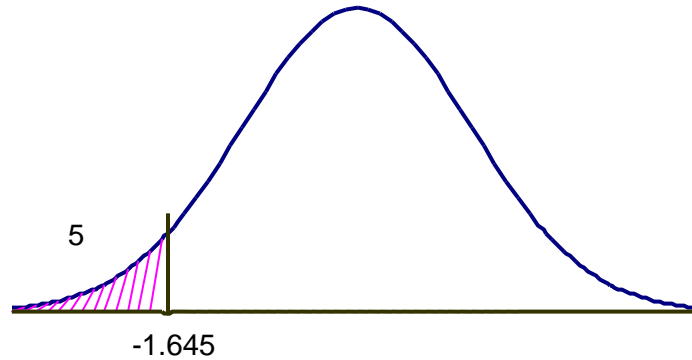
Therefore, reject H₀ at p < .02

PROBLEM 5:

A manufacturer produces drill bits with an intended life of at least 580 hours and a standard deviation of 30 hours. A quality control scientist draws a sample of 100 bits and finds $\bar{X} = 577$. Test at $\alpha = .05$ to see if the machinery needs adjusting.

$$H_0: \mu \geq 580 \text{ hours}$$

$$H_1: \mu < 580 \text{ hours}$$



$$Z = \frac{577 - 580}{\frac{30}{\sqrt{100}}} = \frac{-3}{3} = -1.00 \quad \text{DO NOT REJECT } H_0$$

Do NOT adjust the machinery.

PROBLEM 6:

A researcher samples 100 Dell laptops and finds:

$$\bar{X} = 6.10 \text{ years}$$

$$s = 0.90 \text{ years}$$

(a) Construct a 95% confidence interval

$$6.10 \pm 1.96 \frac{.90}{\sqrt{100}} = 6.10 \pm 1.96(.09) = 6.10 \pm .18$$

$$\mathbf{5.92 \text{ years} \longleftarrow \longrightarrow \mathbf{6.28 \text{ years}}}$$

[If the company uses a confidence interval, they would never claim anything more than 6.28 years.]

PROBLEM 7:

A company that manufactures batteries is interested in constructing a two-sided 95% confidence interval for the population mean.

They sampled 121 batteries and found that the sample mean is 97.0 hours and the sample standard deviation is 3 hours.

$$97.0 \pm 1.96 \frac{3}{\sqrt{121}} = 97.0 \pm 1.96(.27) = 97.0 \pm .53$$

96.47 hours ←————→ **97.53 hours**

PROBLEM 8:

A company that manufactures toasters is interested in constructing a two-sided 98% confidence interval for the population mean.

They sampled 100 toasters and found that the sample mean is 9.40 years and the sample standard deviation is 1.20 years.

The Z-value for a 98% confidence interval is: 2.33 (If you take half of .98, you get .4900 the closest value to .4900 on the Z-table is 2.33.

$$9.40 \pm 2.33 (1.20/\sqrt{100}) = 9.40 \pm .28$$

9.68 years ←————→ **9.12 years**

PROBLEM 9:

A college wants to know the upper limit of a two-sided 95% confidence interval for SAT scores on the Math section (range equals 200 to 800).

They sampled 400 students and found that the sample mean was 550 and the sample standard deviation was 80.

$$550 \pm 1.96 (80/\sqrt{400}) = 550 \pm 7.8$$

542.2 ←————→ 557.8 **ANSWER = 557.8**

PROBLEM 10:

The state of Alabama wants to know the lower limit of a two-sided 90% confidence interval for reading scores of graduating seniors at all high schools in Alabama.

They sampled 900 students and found that the sample mean was 11.30 and the sample standard deviation was 1.50

Note: The Z-value for a 90% confidence interval is exactly midway between 1.64 and 1.65; you can use either 1.645 or 1.65.

$$11.30 \pm 1.645 (1.50/\sqrt{900}) = 11.30 \pm .08$$

$$11.22 \longleftarrow \longrightarrow 11.38$$

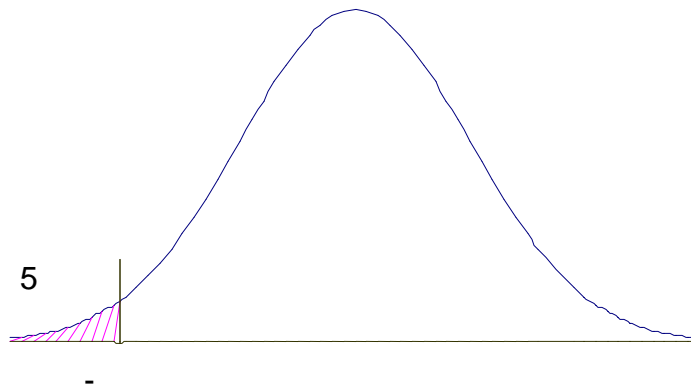
ANSWER = lower limit is 11.22

PROBLEM 11:

A company claims that people using its sleeping pill will sleep for at least 8 hours a night. (a) Use the following sample to test at $\alpha = .05$. (b) suppose no claim was made but the company is thinking of making a claim using a 95% two-sided confidence interval. Construct the interval using the same data and suggest the maximum claim the company can make.

$n=144$
 $\bar{X} = 7.5$ hours
 $s = 1.8$ hours

$H_0: \mu \geq 8$ hours
 $H_1: \mu < 8$ hours



$$Z = \frac{7.5 - 8.0}{1.8 / \sqrt{144}} = \frac{-.50}{.15} = -3.33 \quad \text{REJECT } H_0$$

Therefore, reject H_0 at $p < .05$

(b) For a two-sided 95% confidence interval, the critical Z-value is 1.96

$$7.50 \pm 1.96 (1.8/\sqrt{144}) = 7.50 \pm .29$$

7.21 hours \longleftrightarrow 7.79 hours **ANSWER = upper limit is 7.79 hours The company should not claim more than 7.79 hours using 95% confidence level.**