

SOLUTIONS TO HOMEWORK: PROBABILITY

PROBLEM 1:

Single Status and Microwave Ownership

	S (single)	S' (not single)	
M (own microwave)	40	360	400
M' (no microwave)	60	540	600
	100	900	1000

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<u>Joint Probability</u>	<u>Marginal Totals</u>
P(S and M) = .04	P(S) = .10
P(S' and M) = .36	P(S') = .90
P(S and M') = .06	P(M) = .40
P(S' and M') = .54	P(M') = .60

$$P(M|S) = .04/.10 = .40$$

$$P(M|S') = .36/.90 = .40$$

$$P(M) = .40$$

Therefore, microwave ownership and being single are independent.

PROBLEM 2:

In a study of marriages, researchers examined the faithfulness of men and the survival of the marriage. The results for 5,000 marriages were as follows:

	Faithful (F)	Unfaithful (F')	
Divorce (D)	1300	1200	2500
No Divorce (D')	1700	800	2500
	3000	2000	5000

JOINT PROBABILITY TABLE

	Faithful (F)	Unfaithful (F')	
Divorce (D)	.26	.24	.50
No Divorce (D')	.34	.16	.50
	.60	.40	1.00

a)  $P(D) = .50$

b)  $P(D|F) = 1300 / 3000 = .433$

c)  $P(D \text{ and } F') = .24$

d)  $P(D \text{ or } F) = .50 + .60 - .26 = .84$

e)

Is  $P(D|F) = P(D|F') = P(D)$ ?

$$P(D|F) = .433 \quad P(D|F') = .60 \quad P(D) = .50$$

43.3% of faithful men are divorced vs. 60% of unfaithful men.

Alternate:

Is  $P(D \text{ and } F) = P(D)P(F)$ ?

$$.26 = (.60)(.50) \text{ [NO – not independent – there IS a relationship]}$$

**PROBLEM 3: Tattoos and Marriage**

A study of tattoos on women and marriage found the following:

	Tattoo (T)	No Tattoo (T')	
Married (M)	200	800	1000
Not Married (M')	300	400	700
	500	1200	1700

Compute these probabilities: (a) P (M and T) (b) P (M) (c) P (M/T) (d) P (M or T)  
 (e) P (T/M) (f) Is there a relationship between marriage and having a tattoo?

Note: There is a bit of rounding error.

**JOINT PROBABILITY TABLE**

	T	T'	
M	.118	.471	.589
M'	.176	.235	.411
	.294	.706	1.00

- (a) P (M and T) = .118
- (b) P(M) = .589
- (c) P(M/T) = .118/.294 = .40
- (d) P(M or T ) = .589 + .294 - .118 = .765 (or 1 - .235)
- (e) P (T/M) = .118/ .589 = .20
- (f) P (M) = .589; P(M/T) = .40 (P (M/T') = .667 There is a relationship between having a tattoo and marriage; those without a tattoo, are more likely to be married.

**PROBLEM 4:** (a) what if the probability of being a gum chewer or living in the Northeast? (b) Is there a relationship between region and gum chewing?

	<b>Northeast</b>	<b>South</b>	<b>West</b>
<b>Chews Gum</b>	<b>100</b>	<b>70</b>	<b>130</b>
<b>Does Not Chew Gum</b>	<b>300</b>	<b>230</b>	<b>70</b>

- (a) P (gum chewer OR Northeast) = 300/900 + 400/900 – 100/900 = .67
- (b) P (gum chewer) = 300/900 = .333;  
 P (gum chewer/ Northeast) = 100/400 = .25  
 P (gum chewer/ South) = 70/300 = .233  
 P (gum chewer/ West) = 130/200 = .65  
 There is a relationship; more likely to be a gum chewer if you live in the West

**PROBLEM 5: PREFERENCE FOR CAN**

- (a) What is the probability of preferring a glass bottle? b) What is the probability of preferring a glass bottle *given* that one lives in Region B?  
 c) What is the probability of preferring a can *given* that one lives in Region C?

	<b>Region A</b>	<b>Region B</b>	<b>Region C</b>
<b>Prefer Can</b>	<b>300</b>	<b>190</b>	<b>60</b>
<b>Prefer Glass Bottle</b>	<b>200</b>	<b>110</b>	<b>40</b>

- (a)  $P(\text{prefer glass bottle}) = 350/900 = .389$   
 (b)  $P(\text{prefer glass bottle}/\text{Region B}) = 110/300 = .367$   
 (c)  $P(\text{prefer can}/\text{Region C}) = 60/100 = .60$

**PROBLEM 6: Religion and Divorce**

- (a) What is the probability of being a Protestant? (b) What is the probability of being divorced? (c) What is the probability of being a divorced Catholic? (d) What is the probability of being divorced given that one is Jewish? (e) Is there a relationship between divorce and religion?

	<b>Protestant</b>	<b>Catholic</b>	<b>Jewish</b>
<b>Been Divorced</b>	<b>400</b>	<b>200</b>	<b>100</b>
<b>Never Divorced</b>	<b>600</b>	<b>500</b>	<b>200</b>

- (a)  $P(\text{Protestant}) = 1000/2000 = .50$   
 (b)  $P(\text{divorced}) = 700/2000 = .35$   
 (c)  $P(\text{divorced and Catholic}) = 200/2000 = .10$   
 (d)  $P(\text{divorced}/\text{Jewish}) = 100/300 = .333$   
 (e)  $P(\text{divorced}) = .35$ ;  $P(\text{divorced}/\text{Protestant}) = 400/1000 = .40$ ;  $P(\text{divorced}/\text{Catholic}) = 200/700 = .286$ ;  $P(\text{divorced}/\text{Jewish}) = 100/300 = .333$  There is a relationship; Catholics least likely to be divorced and Protestants are most likely to be divorced

## PROBLEM 7: Smoking and Gender

	M (male)	F (female)	
S (smokes)	150	130	280
S' (doesn't smoke)	250	470	720
	400	600	1000

- (a)  $P(\text{male}) = 400/1000 = .40$   
(b)  $P(\text{female and smoker}) = 130/1000 = .13$   
(c)  $P(\text{female or smoker}) = .60 + .28 - .13 = .75$   
(d)  $P(\text{male/smoker}) = 150/280 = .536$   
(e)  $P(\text{smoker /male}) = 150/400 = .375$   
(f)  $P(\text{smoker}) = .28$ ;  $P(\text{smoker/male}) = .375$ ;  $P(\text{smoker /female}) = .217$  there is a relationship between gender and smoking and men are more likely to be smokers than women.