

SOLUTIONS: TWO-SAMPLE HYPOTHESIS TESTING

Z-TESTS

PROBLEM 1:

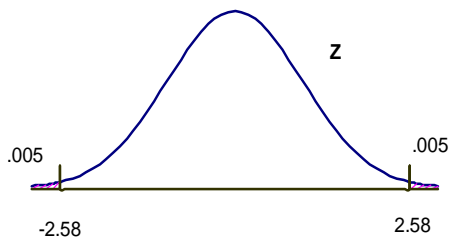
Typing Speed on a pc. Who types faster, Men or Women?

	<u>Men</u>	<u>Women</u>
\bar{X}	65 wpm	68 wpm
s	10 wpm	14 wpm
n	50	60

Test at $\alpha = .01$.

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{65 - 68}{\sqrt{\frac{(10)^2}{50} + \frac{(14)^2}{60}}} = \frac{-3}{2.29} = -1.30 \quad \text{DO NOT REJECT } H_0$$

PROBLEM 2:

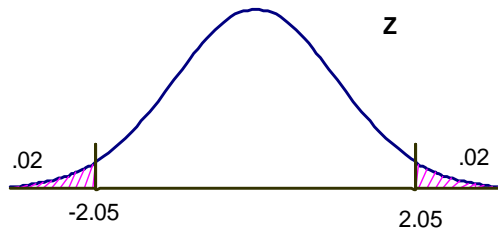
Take-Home Pay. Who earns more: Married or unmarried people?

	<u>Married</u>	<u>Not Married</u>
\bar{X}	\$639.60	\$658.20
s	\$60	\$90
n	40	60

Test at $\alpha = .04$

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{-18.60}{\sqrt{\frac{(60)^2}{40} + \frac{(90)^2}{60}}} = \frac{-18.60}{\sqrt{225}} = \frac{-18.60}{15} = -1.24$$

DO NOT REJECT H_0 .

PROBLEM 3:

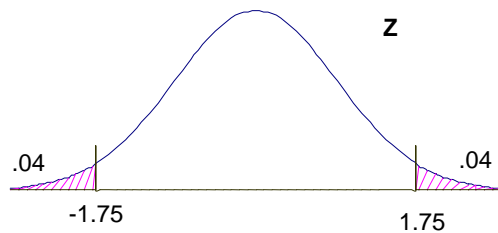
Are the machine tools manufactured by Company X and Y different with regard to how long they last?

	<u>Company X</u>	<u>Company Y</u>
\bar{X}	16.2 weeks	15.9 weeks
s	.2 weeks	.2 weeks
n	40	40

Test at $\alpha = .08$

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{16.2 - 15.9}{\sqrt{\frac{(.2)^2}{40} + \frac{(.2)^2}{40}}} = \frac{.3}{\sqrt{.002}} = \frac{.3}{.045} = 6.67$$

REJECT H_0

PROBLEM 4:

Who lives longer, married or unmarried women? Test at $\alpha = .01$

Single women

$$\bar{X}_1 = 78.5 \text{ years}$$

$$S_1 = 14.0 \text{ years}$$

$$n_1 = 140$$

Married Women

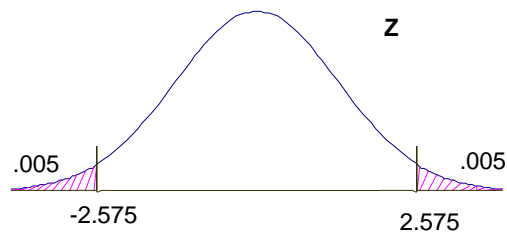
$$\bar{X}_2 = 77.0 \text{ years}$$

$$S_2 = 16.0 \text{ years}$$

$$n_2 = 160$$

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{1.5}{\sqrt{\frac{(14)^2}{140} + \frac{(16)^2}{160}}} = \frac{1.5}{\sqrt{3}} = \frac{1.5}{1.73} = 0.86$$

DO NOT REJECT H_0 .

PROBLEM 5:

Who misses work more often at the ABC Company: Smokers or non-smokers? Test at .05 significance level.

Smokers: Average number of days absent = 14.7; standard deviation = 5.0; n = 44

Non-Smokers: Average number of days absent = 8.3; standard deviation = 4.0; n = 60

$$Z = \frac{6.4}{\sqrt{\frac{(5)^2}{44} + \frac{(4)^2}{60}}} = \frac{6.4}{\sqrt{.834}} = \frac{6.4}{1.73} = 3.70$$

The critical value of Z – two-tail test at .05 significance level is plus and minus 1.96

Reject Ho: 3.70 is greater than 1.96 so we are in the rejection region.
The difference of 6.4 days is statistically significant.

PROBLEM 6:

Who has the higher hourly wage at the ABC Company: Men or Women? Test at .05 significance level.

Men: Average hourly wage = \$12.50; standard deviation = \$1.60; n = 80
Women: Average hourly wage = \$11.40; standard deviation = \$3.20; n = 120

$$Z = \frac{1.10}{\sqrt{\frac{(1.60)^2}{80} + \frac{(3.20)^2}{120}}} = \frac{1.10}{\sqrt{.117}} = \frac{1.10}{.342} = 3.22$$

The critical value of Z – two-tail test at .05 significance level is plus and minus 1.96

Reject Ho: The difference of \$1.10 is statistically significant.

PROBLEM 7:

Who has a longer life span? Test at .05 significance level.

Non-drug user: Average life span = 82.5 years; standard deviation = 12 years; n = 120
Drug user: Average life span = 72.5 years; standard deviation = 12.5 years; n = 50

$$Z = \frac{10}{\sqrt{\frac{(12)^2}{120} + \frac{(12.5)^2}{50}}} = \frac{10}{\sqrt{4.325}} = 4.81$$

The critical value of Z – two-tail test at .05 significance level is plus and minus 1.96

Reject Ho: The difference of 10 years is statistically significant.

PROBLEM 8:

Who earns more? Test at .01 significance level.

High school graduates: Average salary = \$35,000; standard deviation = \$15,000; n = 150

High school dropouts: Average salary = \$26,000; standard deviation = \$10,000; n = 100

$$Z = \frac{9000}{\sqrt{\frac{(15000)^2}{150} + \frac{(10000)^2}{100}}} = \frac{9000}{1581.1} = 5.69$$

The critical value of Z – two-tail test at .01 significance level is plus and minus 2.575

Reject Ho: The difference of \$9,000 is statistically significant.

TWO-SAMPLE T-TESTS

PROBLEM 9:

Who lives longer, married or unmarried men? Test at $\alpha = .01$

Single Men

$$\bar{X}_1 = 72.5 \text{ years}$$

$$S_1 = 7.0 \text{ years}$$

$$n_1 = 14$$

Married Men

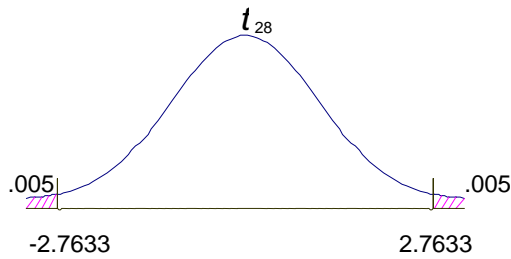
$$\bar{X}_2 = 74.5 \text{ years}$$

$$S_2 = 8.0 \text{ years}$$

$$n_2 = 16$$

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$S_{pooled}^2 = \frac{13(49) + 15(64)}{28} = 57$$

$$t_{28} = \frac{-2}{\sqrt{57\left(\frac{1}{14} + \frac{1}{16}\right)}} = \frac{-2}{\sqrt{7.6}} = \frac{-2}{2.76} = -.72$$

Do not reject H_0 .

PROBLEM 10:

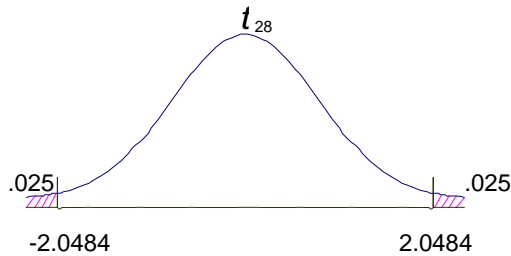
Who does better on the CPA EXAM? Test at .05 significance level

College A: Average score = 70.5; standard deviation = 15.0; n = 12

College B: Average score = 66.5; standard deviation = 10.0; n = 18

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$S_{pooled}^2 = \frac{11(225) + 17(100)}{28} = 149.1$$

$$t_{28} = \frac{4}{\sqrt{149.1\left(\frac{1}{12} + \frac{1}{18}\right)}} = \frac{4}{\sqrt{20.71}} = \frac{4}{4.55} = 0.88$$

Do not reject H_0 . The 4-point difference is not statistically significant.

PROBLEM 11:

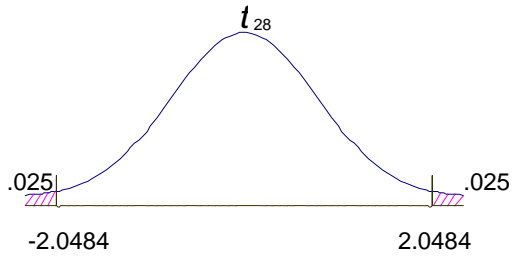
Who misses work more often: male managers or female managers? Test at .05 significance level

Female Managers: Average number of times absent from work = 10.9 days;
standard deviation = 2.4 days; n = 10

Male Managers: Average number of times absent from work = 9.9 days;
standard deviation = 1.8 days; n = 20

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$S_{pooled}^2 = \frac{9(5.76) + 19(3.24)}{28} = 4.05$$

$$t_{28} = \frac{1}{\sqrt{4.05\left(\frac{1}{10} + \frac{1}{20}\right)}} = \frac{1}{\sqrt{.6075}} = \frac{1}{0.78} = 1.28$$

Do not reject H_0 . The 1 day difference is not statistically significant.

PROBLEM 12:

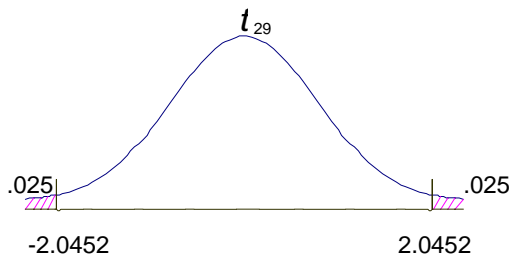
Which yogurt ice cream has less fat? Test at .05 significance level

Company A: Average amount of fat = 10.6 milligrams; standard deviation = 2.6 milligrams; $n = 13$

Company B: Average amount of fat = 10.2 milligrams; standard deviation = 2.4 milligrams; $n = 18$

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$S_{pooled}^2 = \frac{12(6.76) + 17(5.76)}{29} = 6.17$$

$$t = \frac{.4}{\sqrt{6.17\left(\frac{1}{13} + \frac{1}{18}\right)}} = \frac{.4}{\sqrt{.817}} = \frac{.4}{0.90} = 0.44$$

Do not reject H_0 . The .4 milligram difference is not statistically significant.

PROBLEM 13:

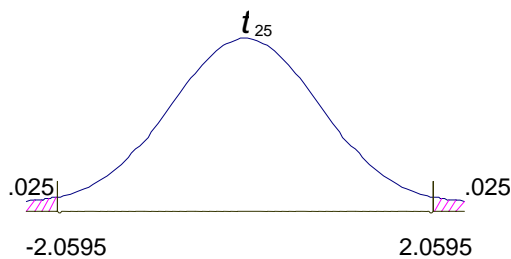
Which approach to treating brain cancer works best? Test at .05 significance level

Approach A: lived on average and additional = 6.8 years; standard deviation = 1.6 years; n = 14 patients

Approach B: lived on average and additional = 5.4 years; standard deviation = 2.4 years; n = 12 patients

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$S_{pooled}^2 = \frac{13(2.56) + 11(5.76)}{24} = 4.03$$

$$t_{25} = \frac{1.4}{\sqrt{4.03\left(\frac{1}{13} + \frac{1}{11}\right)}} = \frac{1.4}{\sqrt{.676}} = \frac{1.4}{0.82} = 1.71$$

Do not reject H_0 . The 1.4 year difference is not statistically significant.

PROBLEM 14:

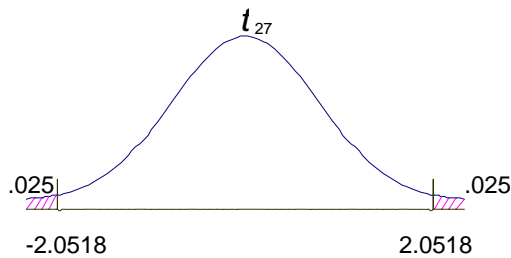
Who makes more money at Company X ? Test at .05 significance level

Men: Average gross income = \$48,500; standard deviation = \$1,000;
n = 18 employees

Women: Average gross income = \$43,600; standard deviation = \$2,000;
n = 11 employees

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$S_{pooled}^2 = \frac{17(1000000) + 10(4000000)}{27} = 2,111,111$$

$$t_{27} = \frac{4900}{\sqrt{2,111,111(\frac{1}{18} + \frac{1}{11})}} = \frac{4900}{\sqrt{309,203}} = \frac{4900}{556} = 8.81$$

Reject H_0 ; The difference of \$4,900 in salary is statistically significant. This firm is paying the men more than the women.

PROBLEM 15:

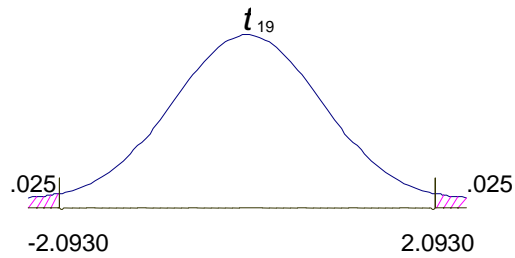
Who makes a better motor ? Test at .05 significance level

Company ABC: Average life= 14.50 years; standard deviation = 1.50 years;
n = 10

Company QQQ: Average life = 13.60 years; standard deviation = 2.10 years;
n = 11

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$



$$S_{pooled}^2 = \frac{9(2.25) + 10(4.41)}{19} = 3.39$$

$$t_{27} = \frac{.90}{\sqrt{3.39\left(\frac{1}{10} + \frac{1}{11}\right)}} = \frac{.90}{\sqrt{.647}} = \frac{.90}{.80} = 1.125$$

Do not Reject H_0 ; The difference of .9 years in motor life is not statistically significant.