# Rules of Probability

1. $0 \leq P(A) \leq 1$

The probability of an event (say, event A) cannot be less than zero or greater than one.

2. $\sum P_i = 1$ or $P(A) + P(A') = 1$

The sum of the probabilities of all possible outcomes (events) of a process (or, experiment) must equal one.

3. Rules of addition.

a. $P(A \text{ or } B) = P(A \cup B)$
$= P(A) + P(B)$ **if** events A and B are mutually exclusive.

Two events are mutually exclusive if they cannot occur together. E.g., male or female; heads or tails.

b. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

This is the general formula for addition of probabilities, for any two events A and B. P(A and B) is the joint probability of A and B occurring together and is equal to zero if they are mutually exclusive (i.e., if they cannot occur together).

4. Rules of multiplication are used for determining joint probabilities, the probability that events A and B will occur together.

a. $P(A \text{ and } B) = P(A \cap B)$
$= P(A)\,P(B)$ **if** events A and B are independent.

Events A and B are independent if knowledge of the occurrence of B has no effect on the probability that A will occur.

b. $P(A \text{ and } B) = P(A|B)\,P(B) = P(B|A)\,P(A)$

This is the general formula for multiplying probabilities, for any two events A and B not necessarily independent.

P(A|B) is a conditional probability. If events A and B are independent, then P(A|B) = P(A), and P(A and B) becomes equal to P(A)P(B), as above.

5. Conditional probability. From the formula in 4.b., we see that we can compute the conditional probability as

$$P(A|B) = P(A \text{ and } B) / P(B)$$

6. Bayes' Theorem.

Since $P(A|B) = P(A \text{ and } B) / P(B)$

and $P(A \text{ and } B) = P(B|A)\, P(A)$

therefore

$P(A|B) = P(B|A)\, P(A) / P(B)$ [We generally need to compute P(B)].