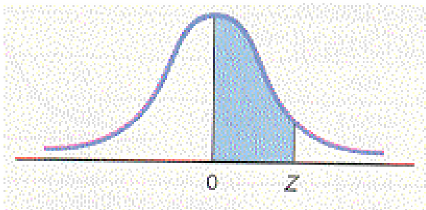
Continuous Probability Distributions

Called a Probability density function. The probability is interpreted as "area under the curve."

1) The random variable takes on an infinite # of values within a given interval

2) the probability that X = any particular value is 0. Consequently, we talk about intervals. The probability is = to the area under the curve.

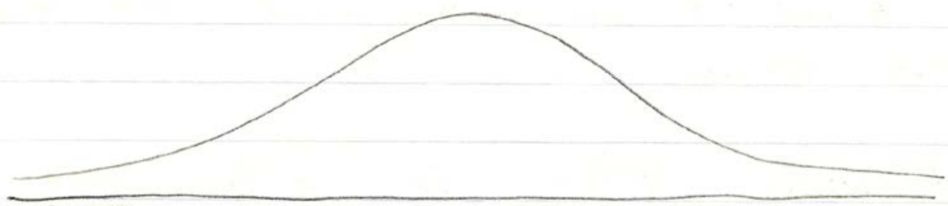
3) The area under the whole curve = 1.



Some continuous probability distributions: Normal distribution, Standard Normal (*Z*) distribution, Student's *t* distribution, Chi-square (χ^2) distribution, *F* distribution.

THE NORMAL DISTRIBUTION

The probability density function for the normal distribution:



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$$

f(X), the height of the curve, represents the relative frequency at which the corresponding values occur.

There are 2 parameters: μ for location and σ for shape.

Probabilities are obtained by getting the area under the curve inside of a particular interval. The area under the curve = the proportion of times under identical (repeated) conditions that a particular range of values will occur.

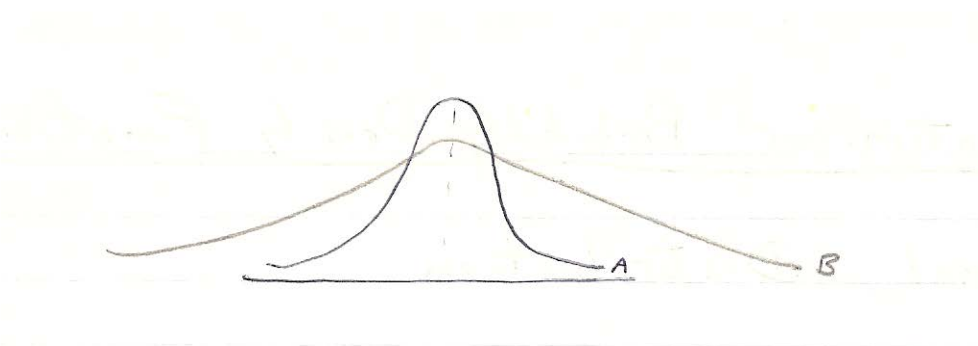
The total area under the curve = 1.

Characteristics of the Normal distribution:
1. it is symmetric about the mean μ.
2. mean = median = mode. ["bell-shaped" curve]
3. f(X) decreases as X gets farther and farther away from the mean. It approaches horizontal axis asymptotically:

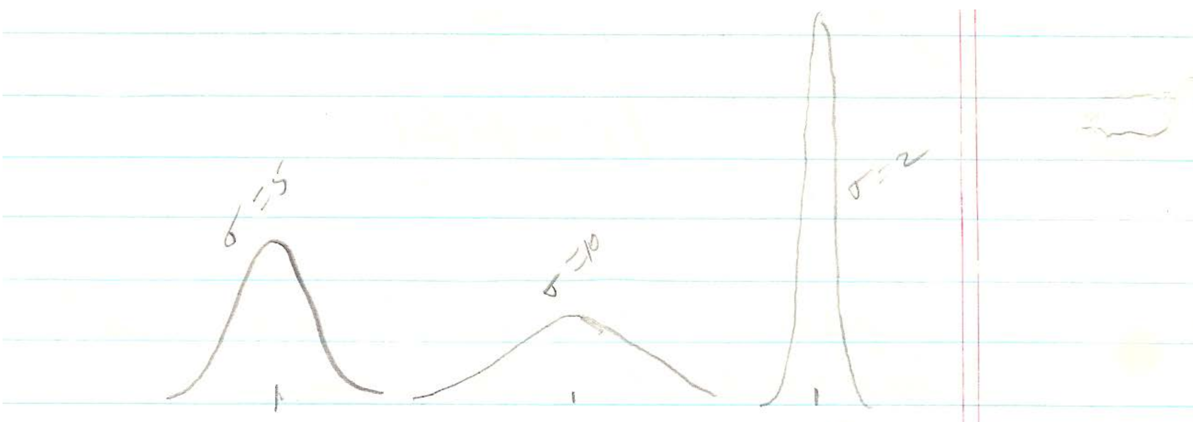
$$-\infty < X < +\infty$$

This means that there is always some probability (area) for extreme values.

Since there are 2 parameters – μ for location and σ for shape. – This means that there are an infinite number of normal curves – even with the same mean.



Curves A and B are both normal distributions. They have the same mean but different standard deviations.



μ=30
σ = 5

μ=80
σ = 10

μ=120
σ = 2

However, there is only ONE Standard Normal Distribution. This distribution has a mean of 0 and a standard deviation of 1.

$$\mu = 0 \qquad \sigma = 1$$

Any normal distribution can be converted into a standard normal distribution by transforming the normal random variable into the standard normal r.v.:

$$Z = \frac{X - \mu}{\sigma}$$

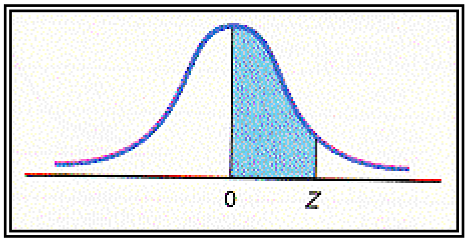
This is called standardizing the data. It will result in (transformed) data with $\mu = 0$ and $\sigma = 1$.

The Standard Normal Distribution (Z) is tabled:

Please note that you may find different tables for the Z-distribution. The table we prefer (below), gives you the area from 0 to Z. Some books provide a slightly different table, one that gives you the area in the tail. If you check the diagram that is usually shown above the table, you can determine which table you have. In the table below, the area from 0 to Z is shaded so you know that you are getting the area from 0 to Z. Also, note that table value can never be more than .5000. The area from 0 to infinity is .5000.

The Normal Distribution is also referred to as the Gaussian Distribution, especially in the field of physics. In the social sciences, it is sometimes called the bell curve because of the way it looks (lucky for us it does not look like a chicken).

THE STANDARDIZED NORMAL (Z) DISTRIBUTION



Entry represents area under the standardized normal distribution from the mean to Z

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.49865	.49869	.49874	.49878	.49882	.49886	.49889	.49893	.49897	.49900
3.1	.49903	.49906	.49910	.49913	.49916	.49918	.49921	.49924	.49926	.49929
3.2	.49931	.49934	.49936	.49938	.49940	.49942	.49944	.49946	.49948	.49950
3.3	.49952	.49953	.49955	.49957	.49958	.49960	.49961	.49962	.49964	.49965
3.4	.49966	.49968	.49969	.49970	.49971	.49972	.49973	.49974	.49975	.49976
3.5	.49977	.49978	.49978	.49979	.49980	.49981	.49981	.49982	.49983	.49983
3.6	.49984	.49985	.49985	.49986	.49986	.49987	.49987	.49988	.49988	.49989
3.7	.49989	.49990	.49990	.49990	.49991	.49991	.49992	.49992	.49992	.49992
3.8	.49993	.49993	.49993	.49994	.49994	.49994	.49994	.49995	.49995	.49995
3.9	.49995	.49995	.49996	.49996	.49996	.49996	.49996	.49996	.49997	.49997

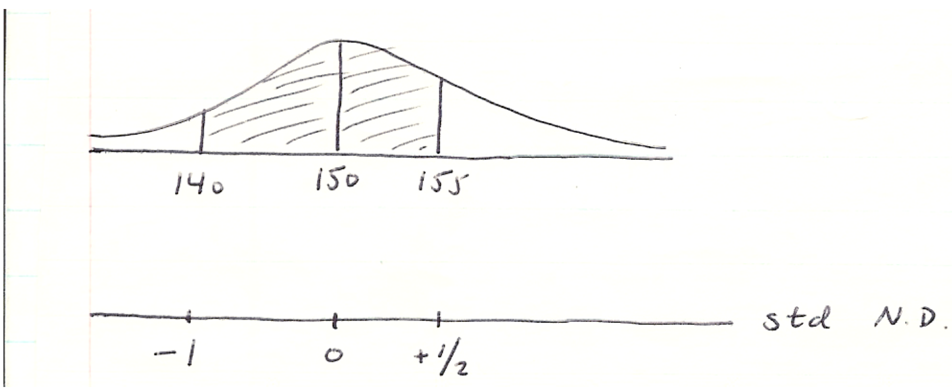
REMEMBER THESE PROBABILITIES (percentages):

# s.d. from the mean	approx area under the normal curve
±1	.68
±1.645	.90
±1.96	.95
±2	.955
±2.575	.99
±3	.997

USING THE NORMAL DISTRIBUTION TABLE

Example:
If the weight of males is N.D. with μ=150 and σ=10, what is the probability that a male will weight between 140 lbs and 155 lbs?

[Important Note:
The probability that X is equal to any one particular value is zero – P(X=value) = 0 since the N.D. is continuous.]



$$Z = \frac{140-150}{10} = \frac{-10}{10} = -1 \text{ s.d. from mean}$$

Area under the curve = .3413 (from Z table)

$$Z = \frac{155-150}{10} = \frac{5}{10} = .5 \text{ s.d. from mean}$$

Area under the curve = .1915 (from Z table)

Answer:
.3413
.1915
.5328 ←

Example:

If IQ is ND with a mean of 100 and a s.d. of 10, what percentage of the population will have
(a) IQs ranging from 90 to 110?
(b) IQs ranging from 80 to 120?

(a) Z = (90 – 100) / 10 = -1   area = .3413
Z = (110 – 100) / 10 = +1   area = <u>.3413</u>
.6826 ←

Answer: 68.26% of the population

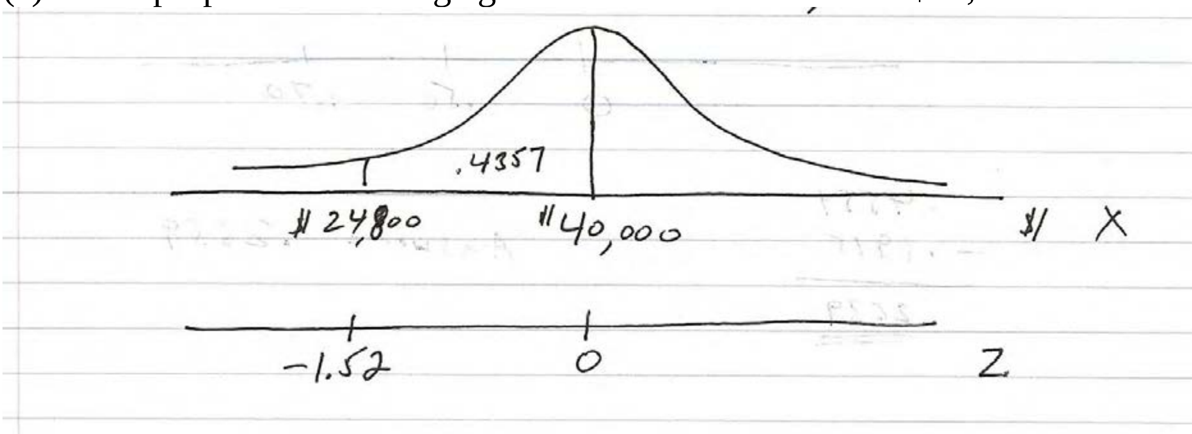
(b) Z = (80 – 100) / 10 = -2   area = .4772
Z = (120 – 100) / 10 = +2   area = <u>.4772</u>
.9544 ←

Answer: 95.44% of the population

Example:

Suppose that the average salary of college graduates is N.D. with μ=$40,000 and σ=$10,000.

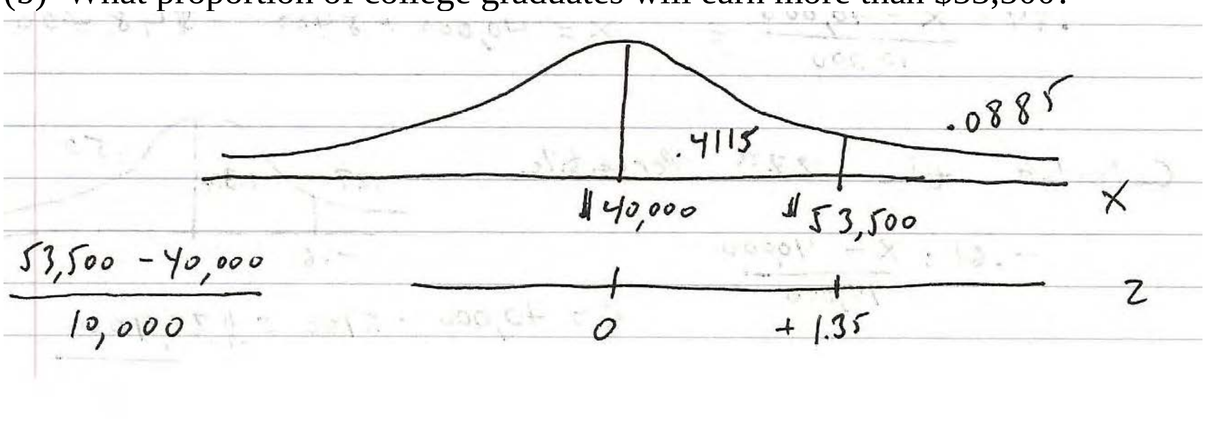
(a) What proportion of college graduates will earn less than $24,800?



Z = ($24,800 – $40,000) / $10,000 = –1.52 area = .0643

➔6.43% of college graduates will earn less than $24,800

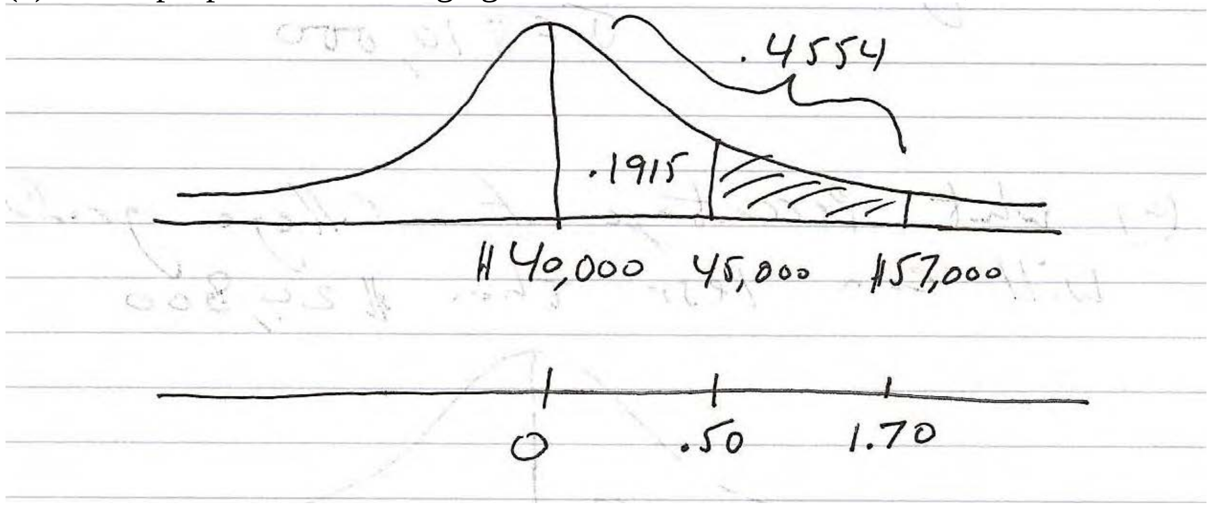
(b) What proportion of college graduates will earn more than $53,500?



Z = ($53,500 – $40,000) / $10,000 = +1.35 area = .0885

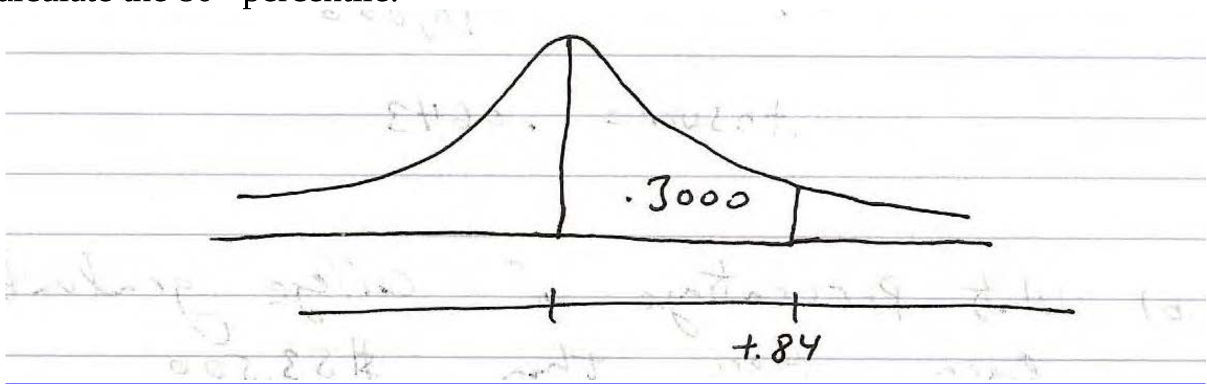
➔8.85% of college graduates will earn more than $53,500

(c) What proportion of college graduates will earn between $45,000 and $57,000?



Z = ($57,000 – $40,000) / $10,000 = +1.70 area = .4554
Z = ($45,000 – $40,000) / $10,000 = +0.50 area = .1915
Answer: .4554 – .1915 = .2639🡐

(d) Calculate the 80th percentile.



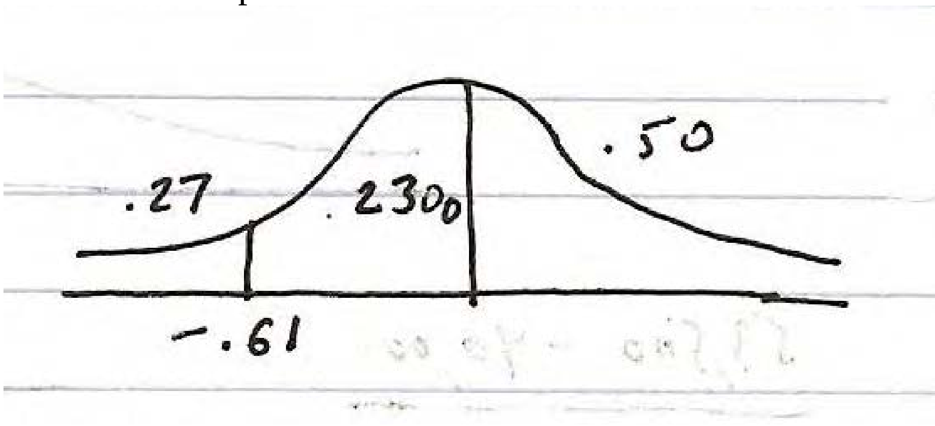
Find the area that corresponds to an area of .3000 from 0 to Z (this means that there will be .2000 in the tail). A Z value of + 0.84 corresponds to the 80th percentile.

+.84 = (X – $40,000) / $10,000

X = $40,000 + $8,400 = $48,400. 🡐

[Incidentally, the 20th percentile would be $40,000 – $8,400 = $31,600]

(e) Calculate the 27th percentile.



Find the area that corresponds to an area of .2300 from 0 to Z (this means that there will be .2700 in the tail). A Z value of – 0.61 corresponds to the 27th percentile.

– 0.61 = (X – $40,000)/ $10,000

X = $40,000 – $6,100 = $33,900 ←

Exercise:

The GPA of college students is ND with μ=2.70 and σ=0.25.
(a) What proportion of students have a GPA between 2.40 and 2.50?
(b) Calculate the 97.5th percentile.
[97.5% of college students have a GPA below _______?]
(c) Calculate the 10th Percentile.
[90% of students will have higher GPAs.]

Answers:

The Z-value for the 2.40 GPA converts to -1.20 [(2.40 – 2.70) / .25];
The Z-value for the 2.50 GPA converts to -.80 [(2.50 – 2.70) / .25];

The area from 0 to -1.20 is .3849
The area from 0 to - .80 is .2881

Answer is .3849 - .2881 = .0968 or 9.68% of college students

(b) A z-score of 1.96 is equal to the 97.5th percentile (.5000 + .4750).

Thus, 1.96 = (X – 2.70) / .25

Solve for X. X = .49 + 2.70 = 3.19 Answer = A GPA of 3.19 is the 97.5th percentile.

(c) A Z score of -1.28 is approximately the 10th percentile.

Find the area that corresponds to an area on the left side (negative) of the Z-distribution of .4000 from 0 to Z (this means that there will be .1000 in the tail). A Z value of -1.28 corresponds to the 10th percentile. A Z-score of + 1.28 is approximately the 90th percentile (actually it is .50 + .3997).

Thus, - 1.28 = (X – 2.70) / .25

Solve for X. X = 2.70 - .32 = 2.38 Answer = GPA of 2.38 is the 10th percentile.

Exercise:

Chains have a mean breaking strength of 200 lbs, σ=20 lbs.
(a) What proportion of chains will have a breaking strength below 180 lbs?
(b) 99% of chains have breaking points below _________? [99th percentile] Hint: 50% have breaking points below 200 lbs which is equal to the population mean. The answer has to be more than 200 lbs. We are on the right side of the Z distribution.

Answers:

Z = (180 – 200) / 20 = -1.00 The area that is between -1.000 and 0 in the Z distribution is .3413. We want the left tail below the -1.000. The entire area to the left of the 0 in the Z-distribution is .5000. Thus,

(a) .5000 - .3413 = .1587 Answer is 15.87%

(b) The value of + 2.33 corresponds to the 99th percentile .5000 + .4901 = .9901. That is close enough for our purposes.

2.33 = (X – 200) / 20

X = 246.60 pounds.

Exercise:
The average life of a stove manufactured by GE is 15 years with a s.d. of 2.5 years.
(a) What percentage of stoves will last 10 years or less?
(b) What percentage of stoves will last 18 years or more?
(c) Calculate the 1^{st} percentile.
(d) Calculate the 96^{th} percentile
(e) What percentage of stoves will last between 16 and 20 years?