TOPIC: PROBABILITY DISTRIBUTIONS

There are two types of random variables:

A Discrete random variable can take on only specified, distinct values.

A Continuous random variable can take on any value within an interval.

DISCRETE PROBABILITY DISTRIBUTIONS

A probability distribution for a discrete random variable is a mutually exclusive listing of all possible numerical outcomes for that random variable, such that a particular probability of occurrence is associated with each outcome.

Probability Distribution for the Toss of a Die:

X_i	$P(X_i)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

This is an example of a uniform distribution.

Discrete Probability Distributions have 3 major properties:

1) $\sum P(X) = 1$
2) $P(X) \geq 0$
3) When you substitute the random variable into the function, you find out the probability that the particular value will occur.

Three major probability distributions:

- Binomial distribution,
- Hypergeometric distribution,
- Poisson distribution.

MATHEMATICAL EXPECTATION

A random variable is a variable whose value is determined by chance.
Expected value is a single average value that summarizes a probability distribution.

$$E(X) = \sum X_i P(X_i)$$

If X is a discrete random variable that takes on the value X_i with probability $P(X_i)$, then the expected value of X – E(X) – is obtained by multiplying each value that random variable X can assume by its probability $P(X_i)$ and summing these products. (In other words, it is a weighted average over all possible outcomes.)

The expected value is normally used as a measure of central tendency for probability distributions (where $\sum P(X_i) = 1$). Hence, $E(X) = \mu$.

$$\mu = E(X) = \sum X_i P(X_i)$$

Example, the probability distribution for the random variable D, the number on the face of a die after a single toss:

D	P(D)	D·P(D)
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	6/6
		21/6

$\mu = E(X) = 21/6 = 3.5$

The expected value is a single average value that summarizes a probability distribution. On average, the value you expect from a toss of a die is 3.5. This is the population mean.

Variance of a random variable:
σ^2 = Var (D) = E[(D_i – μ)^2] = ∑(D_i – μ)^2 P(D_i)

$$\sigma_D^2 = \sum (D_i - \mu)^2 P(D_i)$$
$$= (1-3.5)^2 \cdot 1/6$$
$$+ (2-3.5)^2 \cdot 1/6$$
$$+ (3-3.5)^2 \cdot 1/6$$
$$+ (4-3.5)^2 \cdot 1/6$$
$$+ (5-3.5)^2 \cdot 1/6$$
$$+ (6-3.5)^2 \cdot 1/6$$
$$= 2.9166$$

$$\sigma_D = 1.71$$

Expected Monetary Value

Example:
In the following game, there is an equally likely chance of making $300, $120, and $0. How much would you be willing to pay to play?

V (Dollar Value)	P(V)
$300	1/3
$120	1/3
0	1/3

What is the expected value of this lottery?

E(V) = $140. [$300 (1/3) + $120 (1/3) + $0 (1/3)]. On average, you will make $140 per game if you play the game for a long, long time.

Example:
In the particular game, a coin is tossed. If the coin comes up heads, the player wins $100. If the coin comes up tails, the player loses $50. What is the expected value of the game?

X (Dollar Value)	P(X)	X·P(X)
$100	1/2	$50
-$50	1/2	-$25
		$25

The expected value of this game is $25. Over the long term, this game is worth $25 per toss. If you play this game many, many times (say, 1,000 times) on the average you can expect to make $25 per toss.

Out of, say, 100 tosses, you would expect to win $100 50 times and to lose $50 fifty times. Thus, you will make $2500. This works out to an average winning of $2500 / 100 = $25.

Don’t pay more than $25 to play.

Example:
In the following game, there is a one in 4 chance of winning $80; a one in 4 chance of losing $100; and a one in 2 chance of coming out even. How much would you be willing to pay to play?

V_i (Dollar Value)	$P(V_i)$
-$100	1/4
0	1/2
+$80	1/4

E(V) = -$5 [-$100 (1/4) + $0 (1/2) + $80 (1/4)]

Example: a lottery ticket

How much would you be willing to pay for a lottery ticket with a 1 in 5,000,000 chance of winning $1 million dollars, and a 4 in 5,000,000 chance of winning $100,000?

X	P(X)	X·P(X)
$1,000,000	$\frac{1}{5,000,000}$	.20
$100,000	$\frac{4}{5,000,000}$	.08
$0	$\frac{4,999,995}{5,000,000}$	0
		$0.28

Answer: Don't pay more than 28 cents!

Example:

Would you be willing to pay $9 for a lottery that gives you one chance in a million of making $5,000,000?

V_i (Dollar Value)	$P(V_i)$
$5,000,000	.000001
$0	.999999

The expected value of the above lottery is $5.00. Mathematically, it does not make sense to spend $9 for something that has an expected value of only $5.00. Of course, people do not think this way. Many will spend the $9 or even more for a chance to make $5 million. *Utility theory* is used to explain why people act in this seemingly irrational manner. (This is beyond the scope of this course.)

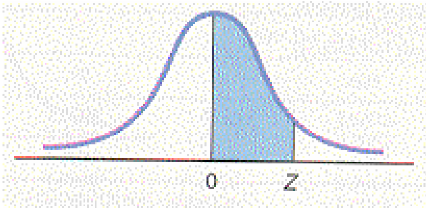
Continuous Probability Distributions

Called a Probability density function. The probability is interpreted as "area under the curve."

1) The random variable takes on an infinite # of values within a given interval

2) the probability that X = any particular value is 0. Consequently, we talk about intervals. The probability is = to the area under the curve.

3) The area under the whole curve = 1.



Some continuous probability distributions: Normal distribution, Standard Normal (*Z*) distribution, Student's *t* distribution, Chi-square (χ^2) distribution, *F* distribution.

NEXT TOPIC: The Normal Distribution