TOPIC: Sample Size Determination

One day some papers catch fire in a wastebasket in the Dean's office. Luckily, a physicist, a chemist, and a statistician happen to be nearby. Naturally, they rush in to help. The physicist whips out a notebook and starts to work on how much energy would have to be removed from the fire in order to stop the combustion. The chemist works on determining which reagent would have to be added to the fire to prevent oxidation. While they are doing this, the statistician is setting fires to all the other wastebaskets in the adjacent offices. "What are you doing?" the Dean demands. To which the statistician replies, "To solve a problem of this magnitude, you need a large sample size."

To determine the sample size we need, we must know desired precision and σ.

$$\overline{X} \pm \overbrace{Z_\alpha \sigma / \sqrt{n}}^{e=\Pr ecision}$$

e is the sampling error

We can use this formula to solve for n:

If $e = \frac{Z\sigma}{\sqrt{n}}$ then $\sqrt{n} = \frac{Z\sigma}{e}$

and so,

$$n = \frac{Z^2\sigma^2}{e^2}$$

EXAMPLE:
Suppose σ=20 based upon previous studies. We would like to estimate the population mean within ±10 of its true value, at α=.05 (i.e., 95% confidence). What sample size should we take?

$$n = \frac{1.96^2 20^2}{10^2} = 15.4$$

so, we need a sample size of at least 15.4, or n=16. We round UP, not down.

Example
Precision = 1; σ = 5
Using 95% confidence interval, what is necessary n?

$$1.96 \times 5/\sqrt{n} = 1$$
$$\sqrt{n} = 1.96 \times 5 = 9.8$$
$$n = 96$$

Example
Precision = .01; σ = 5; CI = 95%

$$1.96 \times 5/\sqrt{n} = .01$$
$$\sqrt{n} = 196 \times 5 = 980$$
$$n = 960400$$

Sample Size Determination for a proportion:

$$n = \frac{Z^2 P(1-P)}{e^2}$$

EXAMPLE:
Suppose we want a maximum error of e = .01 with 95% confidence. Assuming that variance is the highest possible, P=.5

Then,

$$n = \frac{1.96^2 .5(1-.5)}{.01^2} = 9{,}604$$

Let's try that again with e = .03

$$n = \frac{1.96^2 .5(1-.5)}{.03^2} = 1{,}067$$

This is the number that most pollsters work with.