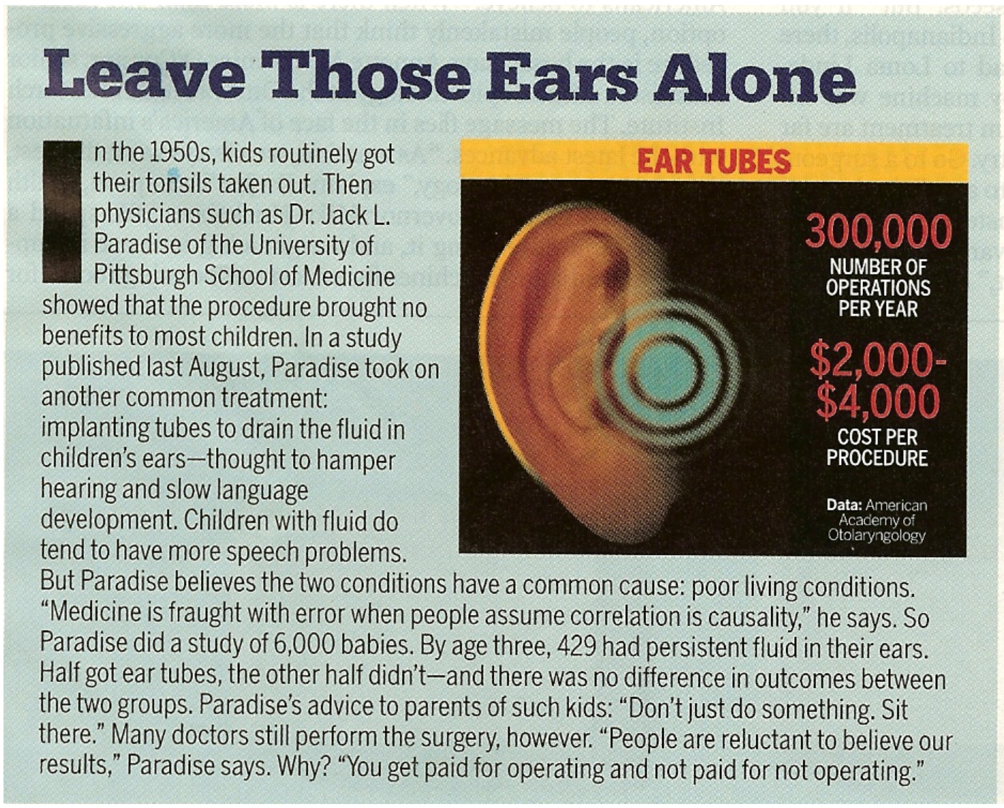
Two Sample (Two Group) Hypothesis Tests



Leave Those Ears Alone

In the 1950s, kids routinely got their tonsils taken out. Then physicians such as Dr. Jack L. Paradise of the University of Pittsburgh School of Medicine showed that the procedure brought no benefits to most children. In a study published last August, Paradise took on another common treatment: implanting tubes to drain the fluid in children's ears—thought to hamper hearing and slow language development. Children with fluid do tend to have more speech problems. But Paradise believes the two conditions have a common cause: poor living conditions. "Medicine is fraught with error when people assume correlation is causality," he says. So Paradise did a study of 6,000 babies. By age three, 429 had persistent fluid in their ears. Half got ear tubes, the other half didn't—and there was no difference in outcomes between the two groups. Paradise's advice to parents of such kids: "Don't just do something. Sit there." Many doctors still perform the surgery, however. "People are reluctant to believe our results," Paradise says. Why? "You get paid for operating and not paid for not operating."

EAR TUBES

300,000 NUMBER OF OPERATIONS PER YEAR

$2,000-$4,000 COST PER PROCEDURE

Data: American Academy of Otolaryngology

Source: Business Week May 29, 2006

The above is a good example of how medical research is conducted. Note that there are two groups: half received ear tubes and the other half did not. Subjects are assigned to the two groups randomly. The researcher was interested in determining whether there was a difference between the two groups in hearing and language development.

EXAMPLE:
Compare the following 2 groups

Drug Group	Placebo Group
$\bar{X}_1 = 4.4$ colds	$\bar{X}_2 = 4.8$ colds
$S_1 = 0.7$ colds	$S_2 = 0.8$ colds
$n_1 = 81$	$n_2 = 64$

Test at $\alpha = .05$ (these tests are usually two-tailed)

Two-Sample Z-Tests

If the samples are large, random, and independent, then $(\overline{X}_1 - \overline{X}_2)$ is a random variable and has approximately a normal distribution, with

$$E(\overline{X}_1 - \overline{X}_2) = \mu_1 - \mu_2$$

and $\sigma_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{\sigma_1^{\ 2}}{n_1} + \frac{\sigma_2^{\ 2}}{n_2}}$

So,
$$Z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^{\ 2}}{n_1} + \frac{\sigma_2^{\ 2}}{n_2}}}$$

However, since H_0 is usually $\mu_1 = \mu_2$, then $\mu_1 - \mu_2 = 0$ and:

$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1^{\ 2}}{n_1} + \frac{\sigma_2^{\ 2}}{n_2}}}$$ for σ_1, σ_2 known

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$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{if } n_1 + n_2 \geq 32$$

For a 1-α % Confidence Interval for the difference between two means:

$$\left(\overline{X}_1 - \overline{X}_2\right) \pm Z_\alpha \sqrt{\frac{\sigma_1^{\ 2}}{n_1} + \frac{\sigma_2^{\ 2}}{n_2}}$$

When we construct the confidence interval for the mean difference in the population **($\mu_1 - \mu_2$),** we check to see whether 0 is in the interval. If 0 is in the interval, then we are basically saying that there may be no difference between the two groups and the observed difference between the two sample means is simply sampling error.

The Null hypothesis that $\mu_1 = \mu_2$, is the same as saying that the hypothesized mean difference is 0, i.e., ($\mu_1 - \mu_2$) = 0.

EXAMPLE:
Compare the following 2 groups

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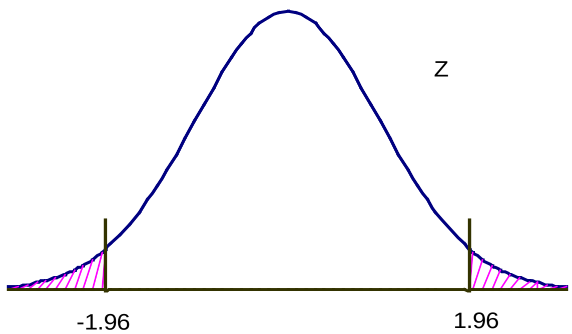
n_1 = 81	n_2 = 64

(a) Test at α = .05 (these tests are usually two-tailed)

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{4.4 - 4.8}{\sqrt{\frac{(.7)^2}{81} + \frac{(.8)^2}{64}}} = \frac{-.4}{\sqrt{.01605}} = \frac{-.4}{.127} = -3.15$$

Therefore, REJECT H_0 $p < .05$. The two groups are indeed statistically different.

(b) Construct a 95% Confidence Interval Estimate for the difference between the two population means

$$-.4 \pm \underbrace{1.96(.127)}_{.25} \Rightarrow \text{-.15} \longleftarrow\!\!\longrightarrow \text{-.65}$$

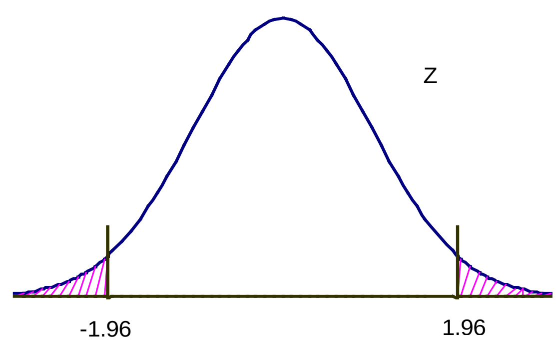
The Drug group has fewer colds. Since 0 is not in the above interval, we see that there is a difference between the two groups. If 0 were in the interval, then we could not reject a 0 mean difference. The drug group has fewer colds. We are 95% sure that the true population mean difference between the drug group and the placebo (control) group is between .15 and .65 fewer colds.

EXAMPLE

Scores on a standardized science test. Is there a difference between men and women? Test at α = .05.

	Men	Women
$\bar{X}$	80.0	76.5
s	10	16
n	100	64

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{80.0 - 76.5}{\sqrt{\frac{(10)^2}{100} + \frac{(16)^2}{64}}} = \frac{3.5}{\sqrt{5}} = 1.56$$

Do not reject H_0 $p > .05$

There is no statistically difference between men and women on test scores in science.

(b) 95% CIE

$$3.5 \pm 1.96\left(\sqrt{5}\right) \Rightarrow 3.5 \pm 4.4$$

-0.9 ←———→ +7.9

↑

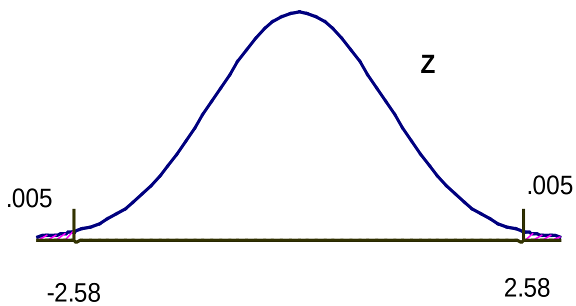
0 difference is possible. When difference goes from negative to positive or positive to negative, 0 is contained in the interval. There may indeed be no difference between men and women on the science test.

EXAMPLE
Typing Speed using MS Word. Who types faster?

	Men	Women
$\bar{X}$	65 wpm	68 wpm
s	10 wpm	14 wpm
n	50	60

Test at $\alpha = .01$.

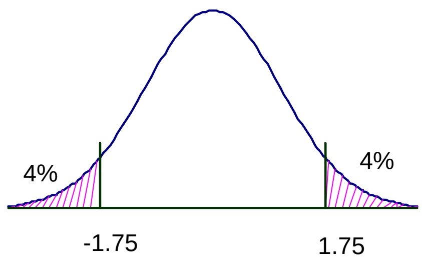
$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{65 - 68}{\sqrt{\frac{(10)^2}{50} + \frac{(14)^2}{60}}} = \frac{-3}{2.29} = -1.30$$

Do not reject H_o. $p > .01$.
There is no statistically difference between men and women in typing speed.

(b) Construct a 92% C.I. for different between the means



$$-3 \pm 1.75(2.29)$$

$$-3 \pm 4$$

-7 ←——→ +1

↑

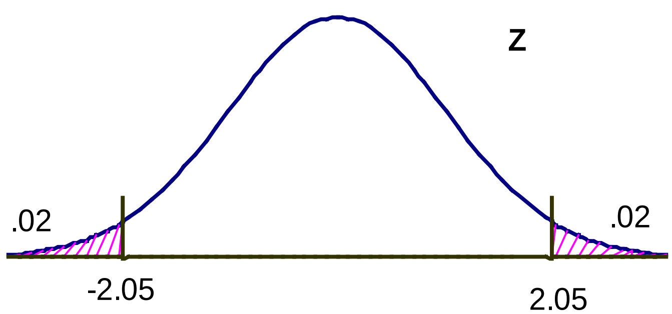
Note that 0 is in the range.

EXAMPLE
Take-Home Pay. Who earns more: Married or unmarried people?

	Married	Not Married
$\overline{X}$	\$639.60	\$658.20
S	\$60	\$90
n	40	60

(a) Test at $\alpha = .04$

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{-18.60}{\sqrt{\frac{(60)^2}{40} + \frac{(90)^2}{60}}} = \frac{-18.60}{\sqrt{225}} = \frac{-18.60}{15} = -1.24$$

DO NOT REJECT H_0. $p > .04$
There is no statistically difference between married and single individuals with regard to take-home pay.

(b) Construct a 95% CIE for the difference between two means

$$-18.60 \pm \underbrace{1.96(15)}_{29.40}$$

-\$48 ⟵——⟶ +\$10.80

↑

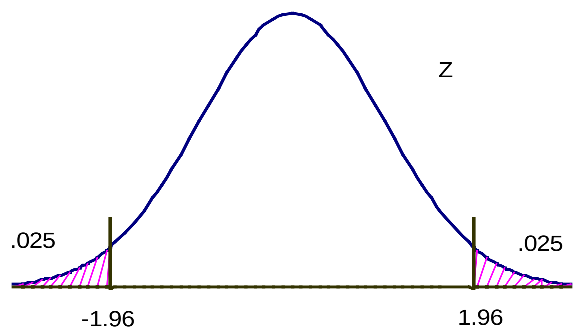
Note that 0 is in the interval.

EXAMPLE
Life Span of Marijuana and non-Marijuana Users

	Non users	Marijuana users
$\overline{X}$	75.2 years	73.2 years
s	8.0 years	7.0 years
n	200	100

(a) Test at $\alpha = .05$

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2$$



$$Z = \frac{75.2 - 73.2}{\sqrt{\frac{(8)^2}{200} + \frac{(7)^2}{100}}} = \frac{2}{\sqrt{.81}} = \frac{2}{.9} = 2.22$$

Reject H_0 $p < .05$. . The two groups are indeed statistically different. Non-users live longer.

(b) Construct a 95% CIE for the difference between the two means

$$2 \pm 1.96(.9) \quad \Rightarrow \quad 2 \pm 1.8$$

$$+.2\,years \longleftrightarrow +3.8\,years$$

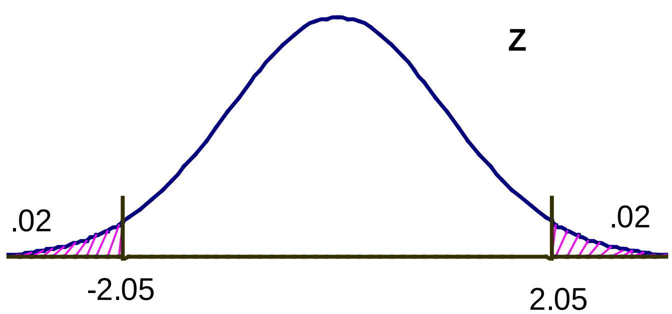
EXAMPLE

Are the machine tools manufactured by Company X and Y different with regard to how long they last?

	Company X	Company Y
$\bar{X}$	16.2 weeks	15.9 weeks
s	.2 weeks	.2 weeks
n	40	40

(a) Test at $\alpha = .04$

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$



$$Z = \frac{16.2 - 15.9}{\sqrt{\frac{(.2)^2}{40} + \frac{(.2)^2}{40}}} = \frac{.3}{\sqrt{.002}} = \frac{.3}{.045} = 6.67$$

Reject H_0 $p<.04$. . The two groups are indeed statistically different. Machine tools manufactured by Company X last longer.

(b) Construct a 92% Confidence Interval for difference

$.3 \pm 1.75(.045) \quad \Rightarrow \quad .3 \pm .08$

$+.22$ weeks $\longleftrightarrow$ $+.38$ weeks